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SCGP student intern Edouard (Eddy) Orsi captured this photo inside the Center for the newsletter. Eddy graduated from Stony Brook University in May 2019 with a major in applied mathematics and a minor in physics. He has been an invaluable assistant creating tools for analyzing and comparing reports and spreadsheets for the SCGP.
EXACTLY SOLVABLE MODELS OF QUANTUM FIELD THEORY AND STATISTICAL MECHANICS
September 4–November 30, 2018
Organized by Nathan Haouzi, Vladimir Korepin, Sergei Lukyanov, Nikita Nekrasov, Samson Shatashvili, Alexander Zamolodchikov

The theory of quantum integrable systems is among the most important achievements in mathematical physics. Its early progress was tied with the study of exactly solvable statistical mechanics models, which led to the discovery of remarkable algebraic structures now commonly known as the Yang-Baxter algebras. The latter gave rise to important developments in quantum integrability, most notably the Quantum Inverse Scattering Method, which has been successfully applied to many interesting problems in condensed matter physics. With the relatively recent discovery of their role in the gauge/string duality and supersymmetric gauge theories, quantum integrable models are also entering the mainstream of high energy theory. Finally, the investigation of the Yang-Baxter algebras and their representation theory has had a profound impact on pure mathematics. The goal of the program was to connect researchers from quantum field theory, condensed matter physics, statistical physics and pure mathematics to discuss the most recent developments in the field of quantum integrable systems. With two talks a week over the course of three months, the program was a resounding success and hosted a variety of lecturers who are world experts in integrability.

In the area of quantum field theory, the talks were mostly focused on TT deformation (flow), ODE/IQFT correspondence, integrable deformations of NLSM, application of QFT methods to condensed matter systems, and highly entangled spin chains. Some participants reported results in the area of pure mathematics; among them, notable lectures were given by P. Bleher, J-M. Stephan, A. Its, T. Dorlas, J. Ellegaard Andersen and A. Stolin. Additionally, key lectures in the area of statistical mechanics were given by A. Kluemper on thermodynamics of exactly solvable spin chains; R. Nepomechie on results on the symmetries of quantum spin chains labeled by an affine Lie algebra; H. Au-Yang Perk on chiral Potts model; H. Katsura on how to entangle spin chains; J. Viti on six vertex models with domain wall boundary conditions; G. Ribeiro on functional Bethe Ansatz; P. Pramod on spin chains with semi-group symmetry; S. Fumihiko on Renyi entropy in Motzkin spin chain; F. Franchini on role of Ising model in quantum information; B. McCoy on correlations of Ising model.

Throughout the program, there was much communication between the participants with regard to their various research projects that could potentially evolve into long-standing collaborations. In particular V. Bazhanov and S. Lukyanov, together with their graduate students G. Kotousov and S. Koval,
initiated a new research project based on the study of a certain staggered spin chain within the framework of the ODE/IQFT correspondence. This model has an interesting potential application to the quantization of a non-compact NLSM. The results of this work are to be published soon with the acknowledgement of the support of the SCGP. V. Bazhanov, G. Kotousov and S. Lukyanov had many fruitful conversations with C. Klimcik on the topic of integrable NLSM. V. Bazhanov, S. Lukyanov and A. Zamolodchikov had numerous discussions centered around the quantum field theory description of disordered systems at criticality. And K. Hao, H. Katsura, G. Ribeiro and V. Korepin started a research project on related highly entangled spin chains [Motzkin, Fredkin and Shor-Movassagh] to Bethe Ansatz. So far, the participants of this research project found a limiting case of Motzkin chain, which can be solved by algebraic Bethe Ansatz.

WORKSHOPS

GEOMETRICAL ASPECTS OF SUPERSYMMETRY
October 22-26, 2018
Organized by Manuela Kulaxizi and Maxim Zabzine

Geometers and high energy theorists came together at this workshop to discuss the fundamental interconnection between supersymmetry and geometry and to celebrate the achievements of Martin Roček.

There were 16 hour-long lectures given by world-renowned experts, including physicists N. Seiberg, C. Vafa, and E. Witten, and mathematicians M. Gualtieri and N. Hitchin. Presented topics included hyperkähler quotients, generalized complex geometry, T-duality, projective superspace, Killing superalgebras, recent advances in F-theory, supersymmetric landscape, and anomalies.

Two out of the five days were specifically devoted to celebrating Martin Roček’s distinguished contributions to the field. Roček co-authored several foundational works in these fields and is not only considered a pioneer, but is also known as a dedicated teacher and mentor. His most frequent collaborator, Ulf Lindstrom, gave a talk on Wednesday evening overviewing Roček’s career and achievements. This talk was well-attended, including approximately 20 of Roček’s students.

DEVELOPMENTS IN QUANTUM FIELD THEORY AND CONDENSED MATTER PHYSICS
November 5-7, 2018
Organized by Zohar Komargodski and Max Metlitski

Recent progress on the dynamics of quantum field theory, a framework that unifies different physics communities, brought together researchers with expertise in high energy physics, including bootstrap, and in condensed matter physics in order to discuss topics of mutual interest and to share new ideas.

Senior speakers from these communities gave talks at this workshop, such as A. Vishwanath, T. Senthil, S. Sachdev, and X.G. Wen from the condensed matter community, and J. Gomis, C. Cordova, F. Benini, and A. Karch from the high energy physics community. New results and the discovery of the different communities attempting to solve the same problems stimulated discussions and forged some new collaborations between attendees.

Topics of discussion included duality, higher symmetries, the phases of three- and four-dimensional gauge theories, conformal field theories in three dimensions and their applications, the classification of anomalies and SPT phases, phases of lattice models, and topological excitations.
NONEQUILIBRIUM PHYSICS IN BIOLOGY
December 3-7, 2018
Organized by Jin Wang, Ken Dill, Michael R. Douglas, Jose Onuchic

Leading researchers from around the world came together at this workshop to discuss current progress in uncovering emergent phenomena and design principles that allow living systems to function, develop, and evolve under nonequilibrium conditions. It was organized to encourage discussion between the diverse research communities of fundamental physical, mathematical, and biological questions in nonequilibrium dynamics across many spatial and temporal scales.

Dubbed by some participants the “Woodstock” or “Solvay Meeting” in nonequilibrium physics in biology, this workshop was the first of its kind at the Simons Center. Its focus on nonequilibrium physics and mathematics in biology set it apart from previous events, namely the Gordon conferences, which focused on stochasticity in biophysics and biology, and the KITP programs and workshops, which focused on nonequilibrium physics.

Workshop participants included 32 speakers, 15 registered participants, and a number of local participants. In addition to workshop talks, there was a discussion session on the current status, challenges, and future direction of nonequilibrium physics and biology. P. Wolynes gave a public lecture entitled Functional Biology: Landscape Physics-A Little Bit Beyond Equilibrium. Research topics included nonequilibrium physics at the molecular and intracellular levels, nonequilibrium physics of the cell structure and dynamics, nonequilibrium physics of evolution and ecology, and nonequilibrium dynamics, thermodynamics, and mathematics.

ENTANGLEMENT AND DYNAMICAL SYSTEMS
December 10-14, 2018
Organized by Israel Klitch, Bruno Nachtergaele, Vladimir Korepin

Studying quantum entanglement has become increasingly important in different branches of science and engineering. The central role of entanglement in a new generation of quantum devices requires further investment in quantum science research and the development of quantum technologies. Quantum science has even become mainstream; the US recently passed the National Quantum Initiative Act, which marks the beginning of a ten-year program to fund such ventures, and MIT’s Technology Review January-February 2019 issue carried an article highlighting potential military applications.

This workshop was devoted to addressing fundamental questions of quantum entanglement and its applications in mathematics, theoretical and experimental physics, chemistry, computer science, and devices. Addressed topics included the solid state approach to building quantum devices, quantum optics, quantum chemistry, rigorous results estimating entanglement entropy in wide class of models, entanglement in string theory, and highly-entangled spin chains.

An important collaboration was initiated during the workshop between O. Salberger, P. Padmanabhan, F. Sugino, A. Trombettoni and V. Korepin. This new project will address an exact solution via the Bethe Ansatz of highly-entangled spin chains.

VERTEX ALGEBRAS AND GAUGE THEORY
December 17-21, 2018
Organized by Sergei Gukov, Tomoyuki Arakawa, Boris Feigin, Alba Grassi, Hiraku Nakajima, Nikita Nekrasov, Andrei Okounkov

Vertex operator algebras (VOAs) have a long history of interaction with physics by describing two-dimensional systems at criticality. Recently, they have appeared via BPS/CFT correspondence in the context of higher-dimensional gauge theories and associated brane constructions in string theory. For example, there are vertex algebras associated to 4-manifolds and to the 4d N=2 superconformal theories. Parallel developments in mathematics led to the construction of the associated varieties of VOAs and geometric realizations of vertex algebras through the generalized cohomology of various moduli spaces.

This workshop aimed to bring together experts in the physics and mathematics of gauge theories and vertex algebras in order to both understand the relationship between these algebraic and geometric structures and to find new ones.

Speakers and participants included T. Arakawa, C. Beem, P. Etingof, D. Gaiotto, M. Kapranov, A. Oblomkov, D. Pei, L. Rastelli, O. Tseytlin and E. Vasserot, among many others. Some topics of discussion included vertex operator algebras and 4-manifolds, 4-manifolds and topological modular forms, Gaudin model and crystals, 3D Kapustin-Rozansky theory and knot homology, higher vertex algebra, the chiral algebra of (0,2) theories in two dimensions, VOAs and 4D SCFTs, and applications of factorization algebras. A colloquium-style public talk was presented by H. Nakajima, entitled Vertex Algebras in 4 Dimensional Theories.
The American Mathematical Society (AMS) awarded two Stony Brook University faculty and one former faculty member with the 2019 Oswald Veblen Prize in Geometry: Xiuxiong Chen, a professor in the Department of Mathematics, Sir Simon Donaldson, a permanent member of the Simons Center for Geometry and Physics and a professor in the Department of Mathematics, and Song Sun, currently in the Department of Mathematics at University of California, Berkeley.

Considered the premier international award in geometry, the Veblen Prize is given for outstanding research work in geometry or topology that has appeared in the past six years. Established in 1961 in memory of Professor Oswald Veblen, it is awarded every three years.

In 1982 Shing-Tung Yau received the Fields Medal in part for his proof of the so-called Calabi Conjecture. He later conjectured that a solution in the case of Fano manifolds, i.e., those with positive first Chern class, would necessarily involve an algebro-geometric notion of stability. The seminal work of Gang Tian, and then Donaldson, clarified and generalized this idea. The resulting conjecture—that a Fano manifold admits a Kähler-Einstein metric if and only if it is K-stable—became one of the most active topics in geometry.

In their three-part breakthrough paper, *Kähler-Einstein metrics on Fano manifolds*, which was published in the Journal of the American Mathematical Society, Professors Chen, Donaldson and Sun proved a remarkable nonlinear Fredholm alternative for the Kähler-Einstein equations on Fano manifolds. According to the Veblen Prize citation, one nominator stated their work “is certainly the biggest result in Kähler geometry since Yau’s solution of the Calabi conjecture 35 years earlier. It is already having a huge impact that will only grow with time.”

Chen received his PhD from the University of Pennsylvania and has been a professor at Stony Brook since 2009. With interests in differential geometry and complex differential geometry, Chen was named a 2015 Fellow of the American Mathematical Society and a 2016 Simons Fellow in Mathematics.

Donaldson, who received his D.Phil. from the University of Oxford, England, joined Stony Brook in 2014. His previous awards include the Junior Whitehead Prize, Fields Medal, Crafoord Prize, King Faisal International Prize, Nemmers Prize in Mathematics, Shaw Prize in Mathematics, and Breakthrough Prize in Mathematics.

Sun joined the faculty at the University of California, Berkeley earlier this year. Previously he was a professor at the Department of Mathematics, Stony Brook University, and held a post-doctoral position at Imperial College, London. He received an Alfred P. Sloan Research Fellowship in 2014, and was an invited speaker at ICM 2018 in Rio de Janeiro.

The Oswald Veblen Prize in Geometry was presented at the 125th Annual Meeting of the AMS in Baltimore, Maryland, in January 2019. The prize was awarded to Xiuxiong Chen, Simon Donaldson and Song Sun.

From left to right: Professor Xiuxiong Chen, PhD; Sir Simon Donaldson, D.Phil; Professor Song Sun, PhD
Photos courtesy Stony Brook University

Canonical Riemannian Metrics and Kähler Geometry

By Dror Varolin
Professor of Mathematics, Stony Brook University

The measurement of lengths, areas and angles is an ancient endeavor, borne out of necessity and inspiring in its depth and flexibility. For example, in their attempt to document angles, some ancient civilizations even divided the circle into 360 equidistant points because 360 is divisible by so many small integers, so from the need to measure, we see the emergence of arithmetic. And closer to now, in the early history of modern mathematics, the discovery by Descartes of coordinates in a plane takes advantage of the measurement of length and direction to turn the geometry of a flat space into a collection of numbers, leading to further developments in arithmetic and algebra.

As the understanding of measurement continued to progress, the algebraic devices used for measurement became simpler to understand and use. One such simple object is the inner product: a bilinear, symmetric, positive definite form on a vector space \( E \). Conveniently, an inner product determines both the lengths of vectors and the angles between them: the length of a vector \( v \) and the (acute) angle between the vectors \( v \) and \( w \) with respect to an inner product \( h : E \times E \to \mathbb{R} \) are respectively

\[
|v|_h := \sqrt{h(v,v)} \quad \text{and} \quad \angle_h(v,w) := \arccos\left(\frac{h(v,w)}{|v|_h |w|_h}\right).
\]

On Earth, the presence of gravity indicates which pairs of vectors ought to be mutually orthogonal, thereby creating a bias in favor of a particular inner product. But, for an abstract vector space, there are many inner products and no canonical way to choose one. Thus, while the set of all inner products \( \mathcal{H}(E) \) of a vector space \( E \) is canonically associated to the vector space, this set has no distinguished point.

Riemannian Metrics

The idea that the notion of orthogonality and length could depend on the point in space at which lengths and angles are measured took much longer to come into existence. Though the idea was present in the work of the preeminent mathematician and scientist Carl Friedrich Gauss (and probably much earlier), the inner products being used were assumed to come from embeddings of surfaces in a vector space. In other words, the perspective was that even though it can contain non-linear objects, the ambient universe is Euclidean. It took the genius of Gauss’ disciple, Georg Friedrich Bernhard Riemann, to free inner products from their ambient jails and view them as objects defined on the tangent spaces of surfaces and higher dimensional manifolds in their own right, assuming only that they vary smoothly as one moves through different tangent spaces. In homage to its creator/discoverer, such a family of inner products is called a Riemannian metric.

Though liberated to vary from tangent space to tangent space, the fact that Riemannian metrics vary smoothly from point to point can have a constraining effect, especially when the space is ‘finite’ (or in mathematical terms, compact). Rather than being completely indistinguishable from one another like their pointwise ancestors the inner products, Riemannian metrics have a strong genetic marker: they have curvature!

Despite its suggestive name, the concept of curvature can be subtle and elusive. The key feature of curvature is that ‘parallelograms don’t necessarily close’. More precisely, given two independent directions \( v \) and \( w \) in \( T_{M,p} \), form the following path: follow the geodesic in the direction of \( v \) for a short time \( T \), parallel translate \( w \) to the vector \( P_T w \) along this geodesic segment, and then follow the geodesic in the direction of \( P_T w \) for a short time \( S \). Now repeat the process with \( v \) and \( w \) and \( S \) and \( T \) interchanged. In flat space these two paths end at the same point, but in a curved space they almost never do. The distance between the two endpoints of this path is in some sense a measure of the curvature.

More precisely, the two endpoints do get rather close together, and if \( S \) and \( T \) are very small then the distance between these two points is of order \( ST \). Since the endpoints of the two paths just constructed are close together, one can connect these two endpoints with a geodesic. One can then parallel translate any vector along the 5-sided path thus obtained. After the full circuit the resulting translated vector, being a length preserving linear transformation of the original vector, will have gone through rotation by a small angle \( \theta_{S,T}(v,w) \). If the path is traversed in the opposite direction, which amounts to interchanging the vectors \( v \) and \( w \), then the angle is approximately replaced by its negative:

\[
\theta_{S,T}(v,w) = -\theta_{T,S}(w,v) + o(S^2 + T^2).
\]
And if $S$ is replaced by $r \cdot S$ and $T$ by $\rho \cdot T$ then

$$\theta_{r \cdot S, \rho \cdot T}(v, w) = r \rho \theta_{S, T}(v, w) + o(r^2 + \rho^2).$$

It follows that

$$\theta(v, w) := \lim_{(S, T) \to (0, 0)} \frac{\theta_{S, T}(v, w)}{ST}$$

is a skew-symmetric bilinear form on $T_{M,p}$. Therefore it is proportional to the oriented area form $dA_h$ associated to the metric $h$. The form $\theta = \theta_h$ is called the curvature form on $h$ and the function of proportionality $K_h$ defined by

$$\theta_h = K_h dA_h$$

is called the Gaussian curvature of the metric $h$. In higher dimensions the curvature also assigns an infinitesimal rotation to a pair of vectors, but the process is somewhat more involved.

**Curvature and Topology.** Mathematics is filled with homonyms: the same word can mean many different things. Geometry is one such mathematical word. The etymology is Greek, and a loose translation is “earth (geo) measurement (metry).” And though the uses in mathematics often have no connection to the earth, most incarnations of the word geometry involve measurement.

By contrast, topology is the study of continuity. If we are given a geometric space – for example, a surface with a Riemannian metric – then the continuous functions on the surface seem to have nothing to do with the metric. If we deform the surface, which just means continuously changing the metric, we tautologically change the geometry, but a function that was continuous before the deformation will remain continuous.

Though deformation changes the geometry, it cannot do so in a completely arbitrary way: the classical mathematical cliché is that the surface of a donut and the surface of a coffee mug are topologically the same, but differ from the surface of a sphere (Fig. 1). It is therefore natural to ask how one can recover the topological object from any of its geometric realizations. In the case of a compact oriented smooth surface without boundary there is a very beautiful answer to the question. The integral of the curvature with respect to the area measure is independent of the metric. In fact, the number

$$\chi(S) := \frac{1}{2\pi} \int_S \theta$$

is an integer, called the Euler characteristic, which completely determines the surface. Formula (1) is a special case of an important theorem called the Gauss-Bonnet Theorem, which determines the Euler characteristic of a compact smooth surface with boundary. There are also higher-dimensional analogues, though they are more complicated and result in more general algebraic objects.

Loosely speaking, averaging of the geometry of a manifold results in something that depends on the manifold alone, and not the particular geometry with which the manifold is endowed.

**Canonical Metrics on Surfaces**

**Canonical Identification of Riemannian Metrics.** The set of all possible Riemannian metrics $\mathcal{R}(M)$ for a given manifold $M$ is, in a vague way, analogous to the set of all inner products $\mathcal{I}(E)$ of a vector space $E$. However, $\mathcal{R}(M)$ is a much larger and more intractable space than $\mathcal{I}(E)$. One might therefore think that, as with inner products, there are no natural ways to pick out a Riemannian metric for a manifold. To a large extent, this is indeed the case, but sometimes when the stars are aligned one can pick out a rather small subset of $\mathcal{R}(M)$. To see one such alignment of the stars, let us look at the historically first case in which this question was treated: the case of compact oriented surfaces.

**Breaking Up $\mathcal{R}(M)$ Into Conformal Classes of Metrics.** An observation going back to Gauss (in the real analytic case) states that given any Riemannian metric $h$ on a surface $S$, there is an atlas of $S$ by coordinate charts with coordinates $(x, y)$ in which the metric takes the form

$$h(x, y) = e^{-\rho(x,y)}(dx^2 + dy^2).$$

Such coordinates are called isothermal (for the metric $h$). In terms of these coordinates, the area form $dA_h$ and curvature form $\theta_h$ determined by the metric $h$ are

$$dA_h = e^{-\rho(x,y)} dx \wedge dy \quad \text{and} \quad \theta_h = \left(\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2}\right) dx \wedge dy.$$

Therefore the Gaussian curvature is given by the formula

$$K_h = e^\rho \left(\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2}\right).$$
Note that although the function $\rho$ depends on the particular isothermal coordinate system, the forms $dA_h$ and $\theta$, and therefore the Gaussian curvature $K_h$, do not.

When two metrics define proportional inner products on every tangent space, i.e., when $\tilde{h} = f \cdot h$ for some smooth function $f : S \to (0, \infty)$, one says that $h$ and $\tilde{h}$ are *conformal*. Thus isothermal coordinates are those in which the metric is locally conformal to the Euclidean metric. If the metrics $h$ and $\tilde{h}$ are conformal then clearly isothermal coordinates for $h$ are isothermal for $\tilde{h}$ and vice versa. Thus the maximal atlas of isothermal coordinates for a metric $h$ provides isothermal coordinates for all metrics conformal to $h$.

In an atlas of isothermal coordinates, the transition functions from one coordinate chart to another send the Euclidean metric to a metric that is conformal to the Euclidean metric. In particular, these transition functions map circles to circles, and if they also preserve the orientation (as we may assume, perhaps after swapping $x$ and $y$) then they are holomorphic. In other words, in the presence of the metric $h$ the oriented surface $S$ becomes a complex manifold of complex dimension 1, i.e., a Riemann surface. Conversely, given a Riemann surface it is easy to construct a Riemannian metric that is locally conformal to the Euclidean metric in every holomorphic coordinate chart. Thus complex manifolds of dimension 1 are in 1-to-1 correspondence with conformal classes of Riemannian metrics.

To go further, we have to make a stronger assumption on the oriented surface $S$. From here on we shall assume that $S$ is compact and without boundary. Such surfaces are always spheres with $g$ handles attached, and the number $g = g(S)$ is called the genus of the surface (Fig. 2). By the way, the aforementioned Euler characteristic is $\chi(S) = 2 - 2g(S)$. If the surface $S$ is a sphere, i.e., $g = 0$, then by making use of basic topology and complex analysis one can show that there is only one possible structure of a complex manifold with which $S$ can be endowed. If the surface $S$ is a torus, i.e., $g = 1$, then again using basic topology and complex analysis one can show that there is a 2-dimensional family of possible complex structures with which $S$ can be endowed. Each of these is represented by one of the complex manifolds $\mathbb{C}/(\mathbb{Z}\omega_1 + \mathbb{Z}\omega_2)$ where $\omega_1$ and $\omega_2$ are complex numbers that are independent as vectors in $\mathbb{R}^2$. The ratio $\alpha = \omega_1/\omega_2$ is then a complex number whose imaginary part is non-zero and any two complex tori with the same $\alpha$ are isomorphic. Not all of the complex tori with distinct $\alpha$ are distinct as complex manifolds. Two ratios $\alpha_1$ and $\alpha_2$ correspond to the same complex torus if and only if there are integers $a, b, c, d$ such that $ad - bc = 1$ and $\alpha_2 = \frac{a\omega_1 + b}{c\omega_1 + d}$. It is easy to see that one can restrict to $\alpha$ lying in the upper half plane, and in this case one shows that two points $\alpha_1$ and $\alpha_2$ of the upper half plane define the same complex torus if and only $\alpha_1$ or $\alpha_1$ can be mapped $\alpha_2$ by Schwarz reflection across finitely many of the blue semicircles in Figure 3.

Thus the space of complex tori can be identified with half of the grey figure after appropriate identification of its sides. This identification shows that the space of tori corresponds to a sphere with three points removed: the vertex, the point $(0,1)$, and the point at $\infty$.

If the surface has genus $g > 1$ then the set of complex structures forms a manifold of dimension $6g - 6^1$. Even a rough explanation requires considerably more work than the cases of genus 0 and 1, so we will not give any form of explanation here.

**Canonical Metric in a Conformal Class.** Let us return to the search for canonical Riemannian metrics for our surface $S$. We have so far stated that the set of conformal classes of metrics on $S$ form finite dimensional sets (with additional structure which we did not discuss in detail). Since the space of Riemannian metrics is obviously infinite dimensional,
being able to canonically choose a metric inside each conformal class would go a long way toward narrowing our choice of metrics. Or looking at it from the point of view of complex geometry, it would be very interesting if on a compact complex manifold of complex dimension 1 a metric could be chosen canonically.

In fact, there is a way to select a canonical class of metrics for a compact complex surface so that for most compact complex surfaces the class contains exactly one metric: one should look for a metric whose curvature is constant. Because of the way that curvature scales with rescaling of the metric, one can assume that the constant is 1, 0 or −1.

As we shall see, metrics of constant Gaussian curvature exist on every compact Riemann surface. In keeping with our classification problem, one wants to know if these metrics are unique. To examine both the existence and uniqueness questions, let us look at the more general problem of prescribing the curvature on a given Riemann surface under the perhaps strong assumption that the sign of the curvature does not change, i.e., the function $K_h$ is either always positive, always negative, or identically zero.

**Flat Metrics.** If the curvature function is identically zero on our surface $S$ then the Gauss-Bonnet formula implies that $S$, having Euler characteristic equal to zero, is a torus. The torus is obtained from the plane by taking a quotient by a group of translations, and since the flat metric in the plane is invariant under translations, any flat metric on the complex plane defines a flat metric on $S$. Thus for a fixed conformal class there is at least a positive ray of flat metrics.

In fact, there are no other flat conformal metrics. Indeed, given a metric $h_0 = e^{-\rho}(dx^2 + dy^2)$, any other metric on the surface is of the form $h_f := fh_0$ for some positive function $f : S \to (0, \infty)$. The curvature of the metric $h_f$ is

$$K_{h_f} = \frac{1}{f} \left( K_{h_0} - e^\rho \left( \frac{\partial^2 \log f}{\partial x^2} + \frac{\partial^2 \log f}{\partial y^2} \right) \right).$$

If both the curvatures $K_{h_f}$ and $K_{h_0}$ are identically zero then $\log f$ is harmonic, and on a compact Riemann surface all harmonic functions are constant.

**Metrics of Negative Curvature.** Next suppose $K$ is a negative function on a surface $S$, and we seek a metric $h_K := e^{-\rho}(dx^2 + dy^2)$ whose curvature is $K$. If $h$ is a metric conformal to $h_K$, i.e., such that $h = fh_K$, and whose curvature is also $K$, then the function $f$ satisfies the equation

$$K = fK + fe^\rho \left( \frac{\partial^2 \log f}{\partial x^2} + \frac{\partial^2 \log f}{\partial y^2} \right).$$

Since the two metrics are assumed to be distinct, the function $f$ is not identically equal to 1. We can assume that there is a point $p \in S$ such that $f(p) > 1$. (Otherwise we can interchange the roles of $h$ and $h_K$, which has the effect of inverting $f$.) Since $K$ is assumed to be negative, we find that at the point at which $f$ achieves its maximum,

$$e^\rho \left( \frac{\partial^2 \log f}{\partial x^2} + \frac{\partial^2 \log f}{\partial y^2} \right) = 1 - \frac{f}{K}$$

is a product of two negative numbers, and hence it is positive. On the other hand, at a point where $f$ (and hence $\log f$) achieves its maximum the Hessian of $\log f$ is non-positive, and therefore so is its trace, i.e.,

$$\left( \frac{\partial^2 \log f}{\partial x^2} + \frac{\partial^2 \log f}{\partial y^2} \right) \leq 0.$$

This contradiction implies that $f \equiv 1$, i.e., that there is at most one metric with curvature $K$.

The existence of a metric of constant negative curvature can be established by appealing to classical complex analysis; the two key ideas are the Schwarz Lemma and the Uniformization Theorem, the latter having been discovered by the great topologist and perhaps last Renaissance mathematician, Henri Poincaré. The existence of these metrics can also be established by solving the partial differential equation, though this is somewhat more involved. Nevertheless, the approach through PDE can yield more: it can be used to construct a metric whose curvature is any given negative function $K$.

**Metrics of Positive Curvature.** If a surface $S$ admits a metric whose curvature is everywhere positive (or
even positive on average), then by Gauss-Bonnet the surface $S$ is topologically a sphere. If we realize $S$ as the unit sphere in 3-dimensional space then the metric inherited from the ambient Euclidean space has constant curvature 1. This metric is often called the round metric. If we pull back the round metric by any conformal (which, in two dimensions, means holomorphic) diffeomorphism of the sphere then we obtain another metric of constant curvature 1. It turns out that this process produces all the metrics of constant curvature 1, i.e., they are all the same as the round metric after a conformal transformation.

The problem of prescribing the curvature of a metric on the sphere was posed by Louis Nirenberg, and has generated an impressive amount of excellent mathematics. The problem is not completely solved, though it was proved by Nirenberg that every sufficiently smooth and everywhere positive function on the sphere is the Gaussian curvature of a metric. The general problem of uniqueness of a given metric of positive curvature has the same solution: two metrics with the same curvature differ by a conformal isometry.

**The Perspective of Calabi.** In the classification problem of the previous section it turned out that the search for a metric of constant curvature results in few metrics. However, the decision to look for metrics of positive curvature seems a little artificial, or perhaps reverse-engineered from one’s knowledge of complex analysis.

A more natural perspective comes about by defining a cost function on the space of metrics in the conformal class of a given metric, or equivalently, the space of metrics on a given Riemann surface $S$ that are locally conformal to the Euclidean metric in any complex chart. The most natural cost function, which we now describe, was discovered by the brilliant and influential geometer Eugenio Calabi. Let us fix our Riemann surface $S$, which we continue to assume is compact without boundary. Denote by $\mathcal{M}(S)$ the set of Riemannian metrics that are locally conformal to the Euclidean metric in every complex coordinate system, and have the additional property that

$$g \in \mathcal{M}(S) \Rightarrow \int_S dA_g = 1.$$ 

For each metric $g \in \mathcal{M}(S)$ we have an area form $dA_g$ on $S$, and we define

$$\mathcal{C}(g) := \int_S K_g^2 dA_g,$$

i.e., the cost of a Riemannian metric $g$ is the square of the $L^2$-norm of its Gaussian curvature with respect to its area form. (The letter $\mathcal{C}$ can suggest ‘cost’ or better yet, pay tribute to ‘Calabi’.)

The Schwarz Inequality

$$\left( \int_S K_g dA_g \right)^2 \leq \left( \int_S dA_g \right) \left( \int_S K_g^2 dA_g \right)$$

shows that if $g \in \mathcal{M}(S)$ then

$$\mathcal{C}(g) \geq (\chi(S))^2.$$

Since the Schwarz inequality is an equality if and only if the functions are proportional, equality is attained if and only if $K_g$ is constant. The space $\mathcal{M}(S)$ is a convex cone inside a rather large vector space whose linear structure is not obvious. The difference $\Theta := dA_{g_1} - dA_{g_2}$ of the area forms of any two metrics $g_1$ and $g_2$ in $\mathcal{M}(S)$ is a differential form whose average over $S$ is zero. The celebrated theorem of Hodge tells us that the form $\Theta$ is therefore exact, but the conformal equivalence of $g_1$ and $g_2$ allows us to go further: there is in fact a function $f$ such that

$$\Theta = \left( \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \right) dx \wedge dy.$$ 

The function $f$ is called the potential of $\Theta$, and any two potentials for the same $\Theta$ differ by a harmonic function, which is constant because the underlying surface is compact. We can therefore normalize the set of potentials, and do so in a number of ways. Perhaps the most natural way to choose a function whose supremum on $S$ is 0.

Conversely, let us fix a metric $g \in \mathcal{M}(S)$. For any smooth function $f$ on $S$ the 2-form

$$dA_g + \left( \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \right) dx \wedge dy$$

is the area form of a metric $g_1$ if and only if

$$\left( \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \right) dx \wedge dy > -dA_g.$$ 

Such functions $f$ are called (strictly) $g$-subharmonic, and they clearly form a cone in the infinite-dimensional subspace $T_{\mathcal{M}(S),g} \subset C^\infty(S)$ of smooth functions on $S$ with supremum 0.

**Higher Dimensions**

The picture of two-dimensional geometry that we developed up to now suggests a number of paths one might take in higher dimensions.
**Geometrization.** In real dimension 3, the next step was taken by Thurston, who supplied the conjectural Geometrization Program and established some of its important results. The final step was taken by Perelman, for which he received, and famously refused, two of the most prestigious prizes in mathematics. This mathematical story is of course well-documented, and we will not retell it here.

**Kähler Geometry.** One can also take the point of view of complex geometry, which we took in the surface case after fixing a conformal class. This approach was initiated by Calabi, who formulated what have come to be known as the Calabi Conjectures. Calabi worked in the setting of Kähler geometry: Kähler manifolds are Riemannian complex manifolds in which the complex and Riemannian geometry coincide in a certain precise sense. Kähler manifolds have many special and interesting properties, but from the point of view of our surface analysis, the most important feature Kähler manifolds possess is that their Ricci curvature, a certain trace of the Riemannian curvature tensor, is so symmetric that it is locally the complex Hessian of a single function. (More precisely, in a Kähler manifold Ricci$(g)$ is the curvature of the Hermitian metric induced on the anti-canonical bundle by the metric $g$.) Practically speaking, the existence of a potential for the Ricci curvature reduces to a single equation the rather hard-to-handle system of PDE that is the analogue of the constant curvature equation for surfaces.

**The Kähler Einstein Problem.** The analogue of the constant curvature equation is the requirement that the Ricci curvature of the metric is proportional to the metric itself. This equation (as well as its more general non-vacuum version) was essentially formulated by Einstein, and its solution is called an Einstein metric. In the setting of complex geometry, the natural problem is to find Kähler Einstein metrics.

An important breakthrough in complex geometry was achieved by Yau, who proved his celebrated and incredibly useful Calabi-Yau Theorem. Aubin and Yau independently worked out the case in which the Ricci curvature is a negative multiple of the metric, showing that as in the surface case, there is a unique metric whose Ricci curvature is the negative of the Kähler form. On any Kähler manifold whose canonical bundle is trivial Yau established the existence of Ricci-flat metrics, and showed that (i) every $(1,1)$-cohomology class that contains a Kähler metric (a.k.a., Kähler class) contains exactly one Ricci flat Kähler metric. Such manifolds are ubiquitous in complex and algebraic geometry, as well as in branches of physics.

As in the surface classification, the case in which the Ricci curvature is a positive multiple of the metric is the hardest to handle. The higher-dimensional analogue of the Gauss-Bonnet Theorem places a strong complex-geometric (and topological) constraint on manifolds that admit such Einstein metrics, forcing them to be so-called Fano manifolds. It has long been known that not every Fano manifold admits a Kähler Einstein metric. Yau conjectured that the existence of an Einstein metric is equivalent to a certain notion of algebro-geometric stability. The formulation of the correct notion of stability, that of $K$-polystability, was first put forth by Tian, and put into its most general form by Donaldson. Tian proved that any Fano manifold with a Kähler Einstein metric is $K$-polystable. But the converse, which became known as the Yau-Tian-Donaldson Conjecture, had resisted proof for a very long time. It was finally solved by Xiuxiong Chen, Simon Donaldson and Song Sun, in a series of three long, but nevertheless dense, papers. The result recently earned Chen, Donaldson and Sun the prestigious Veblen Prize. The work of Chen-Donaldson-Sun has opened up several exciting lines of research, and injected this already active field of research with energy and excitement that continues to inspire activity by many mathematicians in differential, algebroic and complex geometry.

**Calabi’s Extremal Metrics.** Going back to Calabi’s minimal cost point of view, one can also ask for metrics whose scalar curvature minimizes the $L^2$-norm of the scalar curvature of all ‘nearby’ metrics on a given Kähler manifold. Such metrics are called Extremal Kähler metrics, and it turns out that they are exactly those metrics whose scalar curvature is the $(1,0)$-potential of a holomorphic vector field. Since most compact complex manifolds have no holomorphic vector fields, in many cases extremal Kähler metrics are precisely metrics of constant scalar curvature. A special case of these metrics are the Kähler Einstein metrics just discussed. Uniqueness of extremal Kähler metrics, such as it is, was proved by Robert Berman and Bo Berndtsson, but existence remains a problem. An important breakthrough was recently made by Xiuxiong Chen and Jingrui Cheng, again in a series of three papers. But perhaps this is a story for another occasion.
What is Geometry?

Geometry, from the ancient Greek 
geo (earth) and metron (measurement), is often considered a universal quality in human thinking. In fact, this idea of an innate ability to “know” geometry dates back to Plato. In the dialogue Meno, written about 380 BC by Plato, the philosopher Socrates draws out an accurate answer to a geometric puzzle from a young slave who had never studied the subject. With geometry in mind, be it intrinsic or learned, SCGP News invited mathematicians and physicists to share their thoughts on What is Geometry? The only criteria was to provide a candid, perhaps personal perspective, from any angle at any length. ~Lorraine Walsh

DENNIS SULLIVAN

Geometry Derived From Space and Number

The Old Babylonian scribes from around 2000 BC taught their students how to solve problems utilizing lengths, areas, and volumes.

Euclid discovered a path to knowledge by stating explicit self-evident laws to be used in demonstrations of statements about the geometry of space. This is the beginning of modern geometry.

Euclid employed this logical method with his algorithm of division to demonstrate the infinity of prime numbers. Thus also began arithmetic, which relates to geometry.

Descartes invented an explicit coordinate plane and thus introduced real numbers—and hence analysis—into geometry.

Riemann used small right triangles and the infinitesimals of Newton and Leibniz to formulate curvature in models of space in any number of dimensions.

Klein formulated a larger conception of homogeneous geometry in terms of congruences between objects. Congruences can be combined leading to group theory and to Lie theory.

Einstein realized the force of gravity could be interpreted using geometry via Riemann’s curvature of four-dimensional spacetime.

Poincaré simplified geometry, breaking space into benign pieces, whose contiguity properties contained useful and calculable information, beginning the field of topology.

Elie Cartan combined all of the above ideas using parallel transport defined infinitesimally, which added local or gauge symmetry to geometry. This connected to topology via integrands and multivalued functions.

Physicists use observed and conceived geometric symmetries in nature to create models. Gauge symmetry in geometry revealed a richness in the theory of four-dimensional manifolds, Donaldson theory.

Thurston used topology and bounded angle distortion geometry to obtain a complete geometric vision of three-dimensional manifolds, verified by Perelman using a dynamical process on the space of Riemannian geometries.

Gromov created a geometry in the large for groups that gives much insight into the theory of those discrete groups that possess only very thin large triangles. He also created, via an infinite dimensional continuous group, symplectic geometry containing the topological string theory of physics.

Grothendieck, building on Descartes, formulated geometry purely in terms of rings of number-like objects, which can be added and multiplied without regard to order. This geometry is based critically on Euclid’s infinitude of primes and there is a perfect duality between geometry and algebra.

Tate founded a rigid geometry for each prime p motivated by questions that led to the final solution of the Fermat Theorem. Recently, Scholze perfected this setting of Tate, his contribution being like Newton’s addition of limits to classical Greek geometry. This event seems to be the beginning of a new epoch in rigid geometry.

Finally, the discrete or quantum nature of charge plus the reality of space in nature renders plausible the appearance of all of the geometric and arithmetic aspects of mathematics that occur simultaneously in today’s physics models.
JIM SIMONS
What first attracted me to geometry was learning about differential forms, the \( d \) operator, and Stokes theorem. The de Rham theorem came next, and this was soon followed by learning about Lie groups and Lie algebras. I was hooked!

SIMON DONALDSON
A large part of the appeal of geometry, for me, comes from the interaction with what one might call symbolic methods. We have some algebraic formula, say, or a differential equation, which we think about by manipulating symbols on the page. Then we draw a picture and “see” what is going on. The delight is in the interaction between the two ways of thought.

KENJI FUKAYA
The sight is the richest sensation humanity owns. When we try to explain something, we often draw a figure to visualize the contents for an explanation. The role of geometry in mathematics is not so much to give an answer to a problem, since providing an answer to a mathematical question by calculation etc., is usually the role of algebra. The role of the geometer is to say what the answer (often a number) means. Geometry in that sense is not so practical since the answer is given anyway. However, humankind is unable to understand a complicated and chaotic calculation. We need something—a notion, a concept, or a vision—to understand why we calculate that way. That is, I think, the role of geometry in mathematics. Figures appearing in modern mathematics are usually beyond the ability of one’s eyesight. Nevertheless, we visualize important concepts too complicated to understand without geometric meaning. It is amazing how much we (people studying modern mathematics) can learn in our physical eyesight, free from restriction of the human body, as does modern science and technology. The frontier of science and technology requires reaching beyond the border of our imagination, which, I believe, modern geometry can provide.

NIGEL HITCHIN
Virtually every mathematical notion involves some form of spatial image in our mind: the arrow of a map \( f: A \to B \) or an element \( x \) of a set \( A \). Replace these by the words ‘transformation’ or ‘point in a space’ and the abstract becomes more geometrical. Geometrical objects in mathematics carry with them an aura of mental concepts—visual, tactile analogues of the physical world around us—which help us to open up avenues of exploration using whatever mathematical tools are at hand.

JUAN MALDACENA
Einstein has taught us that the force of gravity is due to the geometry of space and time. A geometry that is constrained by Einstein’s equations. A fascinating fact is that the geometry of the universe at large scales is almost flat, but not exactly. It has small deformations that seem to originate from small quantum fluctuations produced at the beginning of the universe. The cosmological shape of our universe is like that of a giant microscope which amplifies small quantum effects to the scale of the whole universe. This fascinating connection between the very small and the very big lies at a very active frontier of current exploration.

CUMRUN VADA
I know of no satisfactory all-encompassing definition of what geometry is, but only examples of what it is. Sufficiently broadly defined it can include all we do in physics and mathematics these days. But perhaps for a narrower definition we can use the age old view of what geometry is: the study of all things that can be measured by a ruler!

JEAN-PIERRE BOURGUIGNON
Editors Note: The following contribution by Jean-Pierre Bourguignon has been edited from an interview:
Concerning the question ‘what is geometry?’—for me, the key characteristic of geometry has been, after a long period of uniqueness, its remarkable capacity to broaden itself several times. One thing I find marvelous and that, actually, lies at the heart of a place like the Simons Center for Geometry and Physics is the perpetual struggle scientists face to understand what is space. This question is critical for physics of course. And the whole point of geometry is to study structures that enrich spaces. So, for me, geometry is the theory of spaces, endowed with all kinds of extraordinary structures.
Spacetime and Quantum Mechanics

By Juan Maldacena

Carl P. Feinberg Professor, School of Natural Sciences
Institute for Advanced Study, Princeton NJ

Juan Maldacena is Carl P. Feinberg Professor at the Institute for Advanced Study in Princeton. After receiving his Ph.D. from Princeton in 1996, Juan Maldacena became associate professor of physics at Harvard in 1997. In November of that year he published his most renowned work, “The Large-N Limit of Superconformal Field Theories and Supergravity.” Maldacena was promoted to Professor at Harvard in 1999 and moved to the Institute for Advanced Study in Princeton in 2001. Among his awards are the MacArthur Fellowship (1999), the APS Bouchet Award (2004), and the Dannie Heineman Prize for Mathematical Physics (2007).

There are few things more basic than geometry: the basic arrangement of points in space, the lines that connect them, etc. Geometry was one of the first subjects that rose from its empirical roots to its formal Euclidean formulation. The notion of geometry is central to physics. In fact, according to Einstein’s theory of relativity, we live in a curved spacetime. Curved and dynamical, spacetime responds to the motion of matter. This was spectacularly confirmed by the discovery, via the LIGO/VIRGO collaboration, of gravitational waves coming from black hole collisions. This discovery also provides the best evidence for the existence of black holes, which are one of the most dramatic consequences of general relativity.

Black holes are regions where the universe is collapsing. Our universe is generally expanding, however in the neighborhood of bound massive objects, our Earth, the galaxy, etc., we can say that it is...
not expanding. In the interior of black holes, it is collapsing. The process that gave birth to the universe at the beginning of the Big Bang is reversed in the black hole interior. The boundary of the collapsing region is called a horizon. A horizon is just an imaginary surface in the sense that an observer that crosses it does not notice anything peculiar. According to classical general relativity, black holes can only grow. Matter can fall in, but once the black hole forms it cannot disappear.

But nature is quantum mechanical, and, as Hawking famously showed, black holes emit thermal radiation once quantum effects are taken into account. This is surprising because, after matter falls in, a black hole is just made out of empty space. Nevertheless, empty space itself can have a temperature. The thermal nature of black holes is related to the fact that black holes have a horizon. The horizon divides the vacuum into two regions: the exterior and the interior. An observer who remains outside has only access to the outside. An interesting property of the vacuum in quantum field theory is that, though it is a single definite state, it cannot be split into single definite states locally. In quantum physics, we call this ‘entanglement’. If you zoom into a small piece of space, the quantum state looks very random, but this randomness is apparent in the sense that random results in that region are correlated with random results somewhere else so that they all fit into what we usually call a ‘pure’ state, or a minimum uncertainty state.

When an observer stays outside a horizon, she is only observing part of the vacuum and that is the reason for its thermal nature. It is possible to quantify how much randomness, or entropy, is generated by the presence of the horizon. It turns out that this is equal to the area of the horizon in Planck units. A Planck unit of distance is the distance at which quantum gravity effects become very important. In nature, it is about $10^{-35}$ meters. With this notion of entropy, and Hawking’s notion of temperature, black holes obey the standard laws of thermodynamics, taking the energy to be equal to its mass (or more precisely $E = mc^2$). In classical general relativity, with ordinary matter, the area of black holes always increases. This is related to the second law of thermodynamics, which says that entropy always increases. These results inspired Hawking to suggest that black holes destroy quantum information. Other researchers suspected that black holes, when they are observed from the outside, behave as ordinary quantum systems that preserve information. In an ordinary quantum system, the increase of entropy is only an approximate notion; it only arises because we are limited to doing only coarse measurements. The system evolves in a complicated way and we lose our ability to predict its behavior because of our limitations on the measurements we do. Through some computations in string theory, there is very convincing evidence this is true. This is based on some computations of black hole entropy from a microscopic point of view, which were done by Strominger and Vafa in 1996. Also, the conjectured duality between gravity in anti-de Sitter space and conformal quantum theories on the boundary implies that black holes behave as relatively standard quantum systems.

In quantum physics there are two types of entropy. One of them is the precise ignorance that we have about the system. For example, we could have a qubit that entangled with another one outside our control. In this case we have some amount of ignorance that is irreducible. No matter how well we measure this qubit, we will not be able to determine its state. This is what we call ‘fine grained’ entropy. On the other hand, we could imagine that we have some set of qubits and we only decide to make coarse measurements. In this case there will be many states that can potentially reproduce our measurements. The number of such states is called the ‘coarse grained’ entropy. This is the entropy that increases under the second law. The fine grained entropy remains constant under evolution. So, the area of the horizon should be interpreted as a coarse grained entropy.

One notable development in black hole physics was a proposal, initially by Ryu and Takayanagi, and then by Hubeny, Rangamani and Takayanagi, of a geometric formula for the fine grained entropy. They proposed that this entropy is computed by the area of an extremal surface in spacetime. As expected, this area is smaller than the area of the horizon and it is a surface that typically lies behind the horizon of the black hole. This is non-zero in cases where the interior of the black hole is geometrically connected to other universes. It is zero in cases where the black hole is produced by gravitational collapse.

This has divided the interior of a black hole into further regions. The region outside the horizon is
the region where we can do measurements in an easy way. The region behind the horizon but outside the extremal surface can be probed by doing more complicated measurements, assuming we have complete control over the black hole degrees of freedom.

An interesting geometry that has been studied intensively in recent years is the full two-sided Schwarzschild geometry. This is actually the solution that Schwarzschild originally found more than a hundred years ago, but maximally analytically extended beyond its coordinate singularities. This was fully done only about fifty years later. But it was understood that it describes two black holes rather than one black hole. In other words, the solution contains two spacetime regions that can be viewed as black hole exteriors and one region that is the interior region, which is common to both black holes. This is a surprising solution in general relativity. If we had such a solution, we would have two objects looking like ordinary black holes from the outside, but they share a single interior. If you and your friend jump into the two distant black holes you could meet immediately in the interior. The spatial distance between the two could be very large in the ambient space, but is very short through the interior. This type of geometry is sometimes called a ‘wormhole’. But it is a wormhole that cannot be crossed. It is a time dependent solution, describing a collapsing region of the universe. You can enter it but, once you enter, you cannot exit it on either of the two sides.

Now, we had said that we can think of a black hole as a quantum system with many degrees of freedom. How should we think about the full Schwarzschild wormhole? In 1976, Werner Israel suggested that it should be viewed as a kind of entangled state, one that arises naturally from a mathematical construction designed to simplify computations in thermal systems. In such cases, it is convenient to double the thermal system, introducing two copies of the original system and setting up an entangled state. So that, when we restrict to measurements on one of the systems, then, from the point of view of that system, we have just a thermal state. These states are called ‘thermofield doubles’. This type of state can, in principle, be constructed for any system. In particular, it can be constructed for the quantum systems that describe black holes. In that case, we expect that the geometry would become connected.

One can wonder how difficult it is to produce such a state. For some systems it does not appear complicated to find a state that is close enough to the thermofield double state. This can be done relatively easily for strongly interacting quantum systems that have some features in common with certain black holes. For this reason, we expect that perhaps there is, at least in principle, some relatively easy way to do it with actual blackholes too.

The interest in this wormhole geometry arises because, in this system, the interior spacetime seems to be arising from entanglement. The most solid and fundamental of notions, spacetime itself, is determined by the ghostly pattern of entanglement in the quantum state. We hope that by studying these issues further we will understand how to think about spacetime in general. In the meantime, this avenue of research has given us some surprising connections. One of my favorites is the connection between quantum teleportation and travel through wormholes, which was shown by Gao, Jafferis and Wall, after some more qualitative suggestions by Susskind. To explain what quantum teleportation is, let us first give a simple classical analog.

Imagine that Alice and Bob want to communicate over radio, with everyone listening, but they do not want other people to understand what they are saying. One way they could do it is to share a secret key. For example, they have two identical copies of a very large random sequence of numbers. So, if Alice wants to send a message, she could convert it into a sequence of numbers and then she can add the numbers (modulo ten) to her copy of the secret key and send the resulting number over the radio. The eavesdroppers would only get a random number and cannot extract the message. On the other hand, when Bob hears the message, he can subtract the numbers from his copy of the key and recover Alice’s original message by taking the received number and subtracting the number in the key (modulo ten).

Quantum teleportation is a quantum version of this. The key is now an entangled state made out of two quantum systems. Alice has one of them and Bob has the other. Now Alice wants to send Bob a quantum state. So, she mixes it very thoroughly with her share of the entangled state and then performs a measurement on the combined system. She sends the result of the measurement over the radio, this
part continues to be classical. Then Bob extracts the quantum state by performing an operation on his share of the entangled state. His operation depends on the result of Alice’s measurement.

One technological application of this process is to send secret messages. One advantage of using quantum mechanics is one cannot copy quantum systems. So, an eavesdropper cannot copy the key without Alice and Bob noticing. On the other hand, for a classical key, there can be many copies so the eavesdropper could have one. This cannot happen in quantum physics because quantum states cannot be copied.

If we take the entangled state to be a wormhole, then one can send a message into the first black hole. By measuring some of the Hawking radiation of this first black hole and sending it to somebody near the second black hole, the second person can send a signal into the second black hole that extracts the message.

Just to explain this in an alternative way, let us imagine a science fiction story. This story takes place sometime in the far future where people can manipulate black holes in a controlled way. In this story, there is a princess and prince Charming. The princess is held hostage in a high tower by her evil stepmother. She allows her to use the internet, even the quantum internet, and she can send quantum messages. The prince is outside and wants to rescue the princess. They read the quantum teleportation paper and come up with an idea to teleport the princess. First, they create lots of entangled qubits and share one of the members with the other person. At the end of the process they have a huge number of entangled qubits. They are getting ready to teleport the princess, and she is setting up the machinery that will mix her with the large number of entangled qubits. But she has second thoughts. She worries that when she gets mixed with all the qubits she will die and it will be painful. She is assured that at the end of the process she will be reconstituted on the other side. However, the process would be painful. But then she reads the paper by Gao, Jafferis and Wall. Thus, she convinces prince Charming to create a black hole with his qubits and she does the same with hers. They further agree to create the black holes in precisely the thermofield double. Now she can jump into her black hole and have the machine collect the Hawking radiation. The evil stepmother thinks she has died in the blackhole. However, the prince receives the result of the measurement and can send the appropriate signal into his black hole. Then something amazing happens. His black hole, instead of emitting Hawking radiation, suddenly spits out the princess, whom he gallantly helps out of the black hole.

The interesting thing is that the princess feels as if she travelled through empty space, and she did not feel anything unpleasant in the process.

From the point of view of the evil stepmother, who stays out of her black hole, she died in the black hole and was mixed very thoroughly with the black hole degrees of freedom.

Notice that the pleasant experience of the princess is due to the emergence of spacetime from the precise entanglement pattern of the two black holes. The idea that spacetime can be manipulated in this way highlights how deep the connection between quantum mechanics and geometry is. The pattern of entanglement generates a particular geometry. We are not studying these questions to rescue princesses, but to understand better the nature of spacetime. Eventually, we would like to better understand the collapsing geometries inside black holes and their big crunch singularities. If that were thoroughly understood, maybe, via an intellectual wormhole, we would also understand the Big Bang singularity...
A Unity of Knowledge

Conversation with Spenta R. Wadia

Interview by Lorraine Walsh

Spenta R. Wadia is Founding Director and Infosys Homi Bhabha Chair Professor at the International Center for Theoretical Sciences (ICTS) of the Tata Institute of Fundamental Research in Bangalore, India. His main research interests are in elementary particle physics, string theory and quantum gravity. His other scientific interests are in statistical mechanics and complex systems including cross-disciplinary biology. He is an alumnus of St. Xavier’s College, Mumbai, IIT-Kanpur, the City University of New York, the University of Chicago and the Tata Institute of Fundamental Research (TIFR) in Mumbai. Below is an excerpt from a spring 2018 interview during Spenta Wadia’s visit to SCGP.

Can you tell me a little about the ICTS? I understand you are the Founding Director.

The International Center for Theoretical Sciences in Bangalore is a center of the Tata Institute of Fundamental Research. The idea to create the ICTS was born in 2001, after the successful completion of the Strings 2001 conference at TIFR Mumbai and a visit to the Infosys Campus in Bangalore in February 2001, that happened at the insistence of Edward Witten who was keen to visit the ‘temples of modern India’. The former boosted our confidence based on our achievement in fundamental physics and the latter provided confidence that institutional infrastructure and management of the highest international quality is possible in India. This combination of the highest quality science within a modern state-of-the-art campus, managed along modern management lines, inspired the basic idea of the ICTS. The idea of ICTS finally took shape in 2004 in discussions with David Gross who gave a concrete road map and impetus to the creation of the Center, which was then established with strong support of C. N. R. Rao by the Tata Institute (of the Government of India) in August 2007. The new state of the art ICTS campus in Bangalore was inaugurated on 20 June 2015.

What is the mission?

ICTS has a three-pronged mission. One mission is of course outstanding research that is organized into three main directions: complex systems that include statistical and condensed matter physics, fluid dynamics and turbulence, and physical biology; spacetime physics that includes gravitational wave astronomy, string theory and related areas; and mathematics that includes geometry, PDEs and probability theory.

At ICTS we see science as one story. The partitioning of science is a result of the history of exploration of the complex natural world and perhaps the limitation of the human mind to comprehend and process and make sense out of so much data. It is good to have staircases built between different areas of science. That is why at ICTS we do not have departments, but units that can interact with each other. We have people who are experts in what they do, but their intellectual
space is broader than their field. And hence they can come together to solve problems that need diverse expertise. An example is the project to understand the mathematical foundations of the Indian monsoon – a weather system, which is very complicated and little understood, given India’s proximity to the equator and being a peninsula. There is also useful interaction between string theorists and condensed matter physicists and mathematicians. At the moment we have approximately 20 faculty members and the number will go up to 30 to 35 in the next 10 to 15 years. Most of them are young and leaders in their fields.

Then the other mission of the ICTS is its ‘Programs’ that bring together physicists, astronomers, cosmologists, mathematicians, biologists, students and researchers from all over the world, under one roof, for various lengths of time to work together to solve the most challenging questions posed by nature, to discover the underlying structures across the sciences and to strive for the unity of knowledge.

The research and quality of the ICTS faculty forms the bedrock of this mission for various activities like schools, conferences, workshops, and discussion meetings. The architecture and spaces of our new campus also contribute to the atmosphere at ICTS. Some of the best people in the world, across the landscape of science, visit and spend time at the ICTS. The ICTS programs also enable a platform for new, important emerging areas of science like the confluence of neuroscience, computer science and artificial intelligence.

Using the resources of our main two missions we have a vibrant program of science outreach and interface with civic society. What we are doing in science is extremely important to share with civic society: with students so they can get interested in science; with people who are not scientists in their professions to have them participate in this incredible journey of our species in the exploration of the natural world and ourselves. Outreach is also very important since we are looking for support from civic society for our work.

During the past decade ICTS has achieved some measure of success in its three missions! Its programs have had a significant impact on Indian science; it is an international hub of science; its faculty (presently 20) has made widely recognised contributions and its science outreach has become a fixture for science enthusiasts in Bangalore. Besides basic government support, it is significant that ICTS activities are presently supported by grants from the Simons Foundation and the Infosys Foundation.

"It is good to have staircases built between different areas of science."

And what does the future hold? How is the transition now that you have stepped down?

In July 2015 I stepped down as Director as I reached the age of 65 years and I handed over the baton to the very capable hands of Rajesh Gopakumar who now leads the ICTS. As for myself, I am happy about the outcome of the years since ICTS began in 2007. Avinash Dhar who has worked side by side with me in all the various tasks since the creation of this Center, continues to be at ICTS. I still work there and continue my research with renewed enthusiasm as I have more time now and also support all initiatives that are currently underway in taking ICTS to an even higher level in its next decade.

Good luck on your new venture. On behalf of everyone at the Simons Center, thank you for sharing your time and work with us. We wish you much success as you move forward. ✪
The following puzzle is contributed by Howard Georgi, Mallinckrodt Professor of Physics at Harvard University.

The solution will be published in the forthcoming SCGP News Volume XIII.

Simple Can Be Harder Than Complex

Here is a standard problem in relativistic kinematics that comes with a puzzle. Suppose that particle 1 with mass $m_1$ and energy $E$ collides elastically with particle 2 with mass $m_2$ at rest, as shown below.

The problem is to find the minimum possible value of $\cos \theta_1$ for $m_1 > m_2$.

The obvious derivation is straightforward, but the answer is MUCH simpler than one might have expected from the intermediate steps, and in fact doesn’t depend on $E$! So the more interesting challenge is to understand why this is true and find a simpler derivation!
The Dainty Professor: The Solution

Answer: The winning strategy is not unique. Here is the optimal strategy that is easy to remember. Reject first $k_0 = \lceil n/2 \rceil$ candidates. Here $n = 100$ is the total number of candidates. After that hire the first second-best-so-far applicant that comes along. The probability of success in this case is: $v_0 = \frac{k_0(n-k_0)}{n(n-1)} = 25/99 \approx 1/4$.

Solution: We refer the reader to the detailed solution of the problem given in: R.J. Vanderbei, “The postdoc variant of the secretary problem.” Here we present only the main steps of the solution.

Let us imagine that the candidate number $k$ enters the room. We can either (i) reject this candidate or (ii) make an offer if the candidate is the second best among all candidates seen so far or (iii) make an offer if the candidate is the best among first $k$ candidates. Let us denote the probabilities of “winning” (winning = offering to the second best among all $n = 100$ candidates) for these cases $v_k$, $g_k$, and $f_k$, respectively, assuming that we are following the optimal strategy. We are interested in finding this optimal strategy and in finding the value of $v_0$.

The probability $g_k$ is the probability that the second best out of first $k$ candidates is also the second best of all $n$ candidates. This can be if the second out of $n$ was assigned one of the first $k$ numbers in line and the very best out of $n$ was assigned one of the remaining $k-1$ first numbers out of $n-1$. We immediately obtain: $g_k = \frac{k}{n} \frac{k-1}{n-1}$. We find $f_k$ in a similar way and obtain: $f_k = \frac{k}{n} \frac{n-k}{n-1}$.

Consider now $k < n$. If the professor declines the candidate $k$ then the probability that

1. the new candidate $k + 1$ will be the best from $k + 1$ is $1/(k + 1)$;

2. the new candidate $k + 1$ will be the second best from $k + 1$ is $1/(k + 1)$;

3. the offer to the new candidate $k + 1$ will be not winning for sure is $(k - 1)/(k + 1)$.

Therefore, we have the following recurrent relation for $v_k$:

$$v_k = \frac{k-1}{k+1} v_{k+1} + \frac{1}{k+1} \max\{v_{k+1}, f_{k+1}\} - \frac{1}{k+1} \max\{v_{k+1}, g_{k+1}\}$$

Now we solve the problem working backwards. We start from $v_n = f_n = 0$, $g_n = 1$ and decrease $k$ from $n$ to 1. We notice that if $v_{k+1} = f_{k+1} < g_{k+1}$, then $v_k = f_k$. The condition $f_k < g_k$ is violated first time when $f_k = g_k$ which gives $k = (n+1)/2$. For $k \geq \lceil (n + 1)/2 \rceil$ we have $v_k = f_k$ and for $k < \lceil (n + 1)/2 \rceil$ the function $v_k$ stops changing. See figure.

The conclusion is that the optimal strategy is: decline first $k_0 = \lceil n/2 \rceil$ and then offer the first candidate who is the second best from the candidates considered so far.
Pondering a Miracle

By Graham Farmelo

Fellow, Churchill College, University of Cambridge, U.K.
Adjunct Professor of Physics, Northeastern University, Boston, MA

Miracle of miracles—that’s how the great physicist Frank Yang described the discovery that some of the beautiful structures of modern mathematics precisely describe our universe’s underlying order. This link between the concrete world of physics and the abstractions of mathematics is at the heart of some of the most exciting research now being done at the overlap between the two worlds.

Yang, a Nobel-prize winner, was on the Faculty at Stony Brook University from 1966 to 1999. During that time, he did path-breaking research on the relationship between mathematics and physics. In the early 1970s, after an especially enlightening discussion with Jim Simons, head of Stony Brook’s mathematics department, Yang made one of his impressive leaps of imagination. He identified several connections between advanced concepts long familiar to geometers and the well-established gauge theory of sub-atomic particles that he and physicist Robert Mills had proposed about fifteen years earlier. The connection between gauge theories and geometry became a vibrant research topic, especially after a series of brilliant papers by the mathematician Michael Atiyah and his colleagues. Atiyah later said that he had morphed into a "quasi-physicist."

One of the key lessons of twentieth-century science—that geometry is central to our understanding of fundamental physics—was soon clearer than ever. It is fitting that the charitable foundation established in 1994 by Jim Simons and his wife Marilyn later made it possible to set up at Stony Brook the Simons Center, now one of the world’s leading centers of research into topics in geometry and physics, which are inextricably connected in ways that continue to puzzle experts.

I first visited the Simons Center for Geometry and Physics one summer morning in 2016, mainly to talk with the mathematician Simon Donaldson. My purpose was to seek his advice about topics covered in a book I was researching on the relationship between cutting-edge pure mathematics and the search for the most fundamental laws of nature. Although I was
at the Center for only a day, I saw that it is an embodiment of the key message of my book, titled *The Universe Speaks in Numbers*: by working alongside each other and even collaborating, physicists and mathematicians can profoundly enrich their understanding of both subjects. The symbiotic relationship between the subject was expressed crisply by the British theoretical physicist Paul Dirac in his lecture *On the Relation between Physics and Mathematics*, which he delivered in Edinburgh eighty years ago. He noted: “as time goes on it becomes increasingly clear that the rules that the mathematician finds interesting are the same as the ones that Nature has chosen.” When I showed extracts of that lecture to Donaldson, he seemed—like many contemporary mathematicians and physicists—to be taken aback by the clarity and far-sightedness of Dirac’s vision.

Dirac would have loved Donaldson’s work, especially the extraordinarily rich insights Donaldson gave into four-dimensional manifolds—loosely speaking, four-dimensional spaces—using Yang and Mills gauge theory. This advance wonderfully exemplifies how an idea generated in the course of physics research can help advance the boundaries of mathematics. Michael Atiyah told me a few weeks before he died last January that Donaldson’s work on four-dimensional manifolds was “the most important development in mathematics in the last quarter of the twentieth century.”

Donaldson later made many other contributions that turned out to be extremely useful to physicists, especially to gauge theorists. This is an excellent example of the truth expressed by a catchphrase now often used by mathematicians and physicists: not only is mathematics unreasonably effective in physics—as the theoretician Eugene Wigner memorably remarked in 1959—physics is also unreasonably effective in mathematics—as the theoretician Eugene Wigner memorably remarked in 1959—physics is also unreasonably effective in mathematics.

Last November, I returned to the Simons Center for a week to give a public lecture about Dirac and view that mathematical beauty is the lodestar for research into fundamental theoretical physics. It was a privilege for me to have the opportunity to meet some of the researchers based at the Center—mathematicians, physicists and the specialists sometimes known as physical mathematicians (who seek to shed light on both new mathematics and the laws of Nature). I was especially pleased to record interviews with Simon Donaldson, the physicist Zohar Komargodski and the Center’s director Luis Álvarez-Gaumé. These recordings will shortly be released online as part of the series that explores some of the themes of my new book, and include discussions with several other leading figures in the field, including Edward Witten, Nima Arkani-Hamed and Greg Moore. Research at the Simons Center is very much in the interdisciplinary tradition that dates back at least to the publication of the *Principia* in 1687, written by Isaac Newton. A precursor of today’s theoreticians, he was a consummate mathematician and a peerless thinker about the inner workings of the universe, not to mention a world-class experimenter.

At the end of my stay, Luis Álvarez-Gaumé invited me to return for a longer visit, and I eagerly accepted. I look forward to witnessing the progress made by the mathematicians and physicists at the Simons Center as they help to explore Yang’s “miracle of miracles”—and, perhaps one day, to understand it. Or is it, as Einstein suggested three years before he died, “an eternal mystery?”

Graham Farmelo is author of *The Universe Speaks in Numbers*, published by Basic Books. The interviews that complement the book will be published in the News section of www.grahamfarmelo.com.
The Simons Center Art and Science Program

By Lorraine Walsh

Art Director and Curator of SCGP and Visiting Associate Professor of Art

EXHIBITIONS

“Architecture is a science arising out of many other sciences,”¹ wrote the great Roman architect Vitruvius in his treatise De architectura. He believed architects should be versed in many disciplines, including mathematics, astronomy, optics, history, philosophy, music, theatre, medicine and even law.

Vitruvius’s interdisciplinary approach is found today in the work of architect, designer and artist Jenny E. Sabin. Her work is a fusion of science and mathematics. Informed by fine art aesthetics and new media, Sabin utilizes data computation, responsive materials and sustainable technologies to apply insights and theories from biology and geometry to structural design.

The Simons Center was honored to host the first survey of Sabin’s multi-faceted work in Matter Design Computation: Projects 2012-2018: An exhibition of drawings and artifacts by Jenny E. Sabin and Jenny Sabin Studio. Featuring a six-year span of technological innovation, eight new drawings by Sabin were showcased with five maquettes in the Simons Center Gallery last fall. Notably on display was a section of the award-winning installation Lumen, which was exhibited at MoMA PS1 in New York City in 2017. (See sidebar for more information.)

At its core, Sabin’s architecture investigates the visualization and simulation of complex spatial datasets intertwined with traditional craft. She fabricates and produces a diverse—if not dazzling—array of material systems and code. This includes robotically woven, knitted and braided textiles, rapid prototyping and 3D printed ceramics, bio-plastics and hydrogels, water-jet cut metals, and a host of software and computer languages. Through productive ‘tinkering’ and repurposing digital fabrication machinery, Sabin’s research yields bio-inspired material systems and software design tools for the assemblage of cross-disciplinary and otherworldly environments.

Sabin takes risks. Her experimental work dares to fuel time-honored craftsmanship with digital technology. And it is with confident collaboration across

disciplines a fresh architecture is built. It is at once transformative, interactive and socially engaging.

To provide some history about Sabin’s luminous career, she is an architectural designer working at the forefront of a new direction for 21st century architectural practice. And as the Arthur L. and Isabel B. Wiesenberger Professor in Architecture and Director of Graduate Studies in the Department of Architecture at Cornell University, Sabin established a new advanced research degree in *Matter Design Computation*. She is also principal of Jenny Sabin Studio, an experimental architectural design studio based in Ithaca, and Director of the Sabin Lab at Cornell AAP. Sabin holds degrees in ceramics and interdisciplinary visual art from the University of Washington and a Master of Architecture from the University of Pennsylvania. She was awarded a Pew Fellowship in the Arts 2010 and was named a USA Knight Fellow in Architecture. In 2014, she was awarded the prestigious Architectural League Prize. Her work has been exhibited internationally in venues including the FRAC Centre, Cooper Hewitt Design Triennial, and recently as part of Imprimer Le Monde at the Centre Pompidou. Her book *LabStudio: Design Research Between Architecture and Biology*, co-authored with Peter Lloyd Jones, was published in 2017. That same year Sabin won MoMA & MoMA PS1’s Young Architects Program with her submission *Lumen*.

While the end goals may differ in science and architecture, there is a driving necessity in both disciplines to spatialize, organize, contextualize, model, and fabricate complex, emergent, and self-organized nonlinear systems.²

*Lumen by Jenny Sabin Studio*

Exhibited at MoMA PS1, New York
June 29–September 4, 2017

*Lumen* was the winner of the esteemed 2017 Museum of Modern Art’s PS1 Young Architect Program (YAP), an award for emerging architectural talent to develop innovative structures for temporary outdoor installation at MoMA PS1 during the summer months. Debuting last year, *Lumen*’s web-like structure provided respite for visitors with water, shade and seating under a canopy of fabric with hexagonal shapes and solar-active material.

*Lumen*’s responsive design is made with more than 1,000,000 yards of digitally knitted fiber and 100 robotically woven spool chairs. An interactive lighting system is also rigged to the canopies, incorporating 250 tubular structures that emit an ethereal glow at night. Also integrated into the architecture is a misting system that creates microclimates, so to speak, which respond to the visitor’s proximity by activating sensors in fabric stalactite shapes that spritz a refreshing mist.

Held in tension within the PS1 courtyard matrix of walls, *Lumen* was informed by biology, materials science, mathematics and engineering. Material response to sunlight and physical participation were vital parts of the studio’s exploratory approach to the high tech textiles for the production of this transformative and adaptive environment. The architecture was thus mathematically generated through form-finding simulations informed by sunlight, the site itself and new materials, to build a structural morphology of knitted cellular components.

Photo: Maxine Hicks

(Above) *Lumen*, MoMA PS1, NY
Photo: Pablo Enriquez
The Sense of Beauty

SCIENCE AND ART FILMS
A different kind of architecture is found at CERN, home to the largest and most powerful particle accelerator in the world: the Large Hadron Collider (LHC). Located just outside of Geneva, scientists here look for the building blocks of the universe, rendering the invisible visible with complex data and interconnected machinery. It is within this infrastructure for high-energy physics research the filmmaker Valerie Jalongo found inspiration for his film The Sense of Beauty.

Screened at the Simons Center in fall 2018, The Sense of Beauty takes the viewer on a journey through CERN’s underground laboratory and considers how, and if, beauty and harmony are inspiring principles for scientists and artists. The immense machines at CERN capture images with the same mysterious energy found in the aesthetics of artists such as Olafur Eliasson, Michael Hoch, Carla Scaletti, and many others.

The film reveals the spirit of humankind’s relationship with science and nature in subtleness. Through candid conversation, one considers the great questions of philosophers and mystics about the world, our origin and destiny. Although individual views and beliefs may differ, all recognize there is an ineffable meaning of beauty when trying to grasp the nature of matter, the cosmos, and existence itself.

“When I'm working on a problem, I never think about beauty. I think only how to solve the problem. But when I have finished, if the solution is not beautiful, I know it is wrong.”

R. Buckminster Fuller

The Sense of Beauty panelists and Luis Álvarez-Gaumé
Photo: Edouard Orsi
The film was followed by a panel discussion titled *Beauty and Science: Encounters and Conversations*. Distinguished Stony Brook University faculty representing five disciplines: Brooke Belisle (Art History), Robert P. Crease (Philosophy), Sir Simon Donaldson (Mathematics), Nobuho Nagasawa (Art), and Peter van Nieuwenhuizen (Physics) engaged in an intriguing, if not elusive, dialogue on beauty and science.

Director Luis Álvarez-Gaumé opened the conversation with an apropos analogy to Raphael’s iconic painting *The School of Athens*; a gathering of the greatest thinkers from different times to discuss truth and beauty, among many other things. Álvarez-Gaumé suggested the film offers an invitation to meditate on these ideas, enticing questions that may have no answers. That is, although the meaning of beauty, truth, art and science all have different import throughout time, they remain true to one notion: the unanswerable. And as the panelists adeptly proved, it is in the questioning one finds weight and significance.

The distinct responses by the panelists were refreshingly diverse. The discussion ranged from ways science and nature communicate to how science is perceived through machinery versus the human eye. Which provides more truth? They considered it was not just a question of how science unveils the beauty of nature, but how we are limited in our interpretation. Is it through the eye or the brain? Some suggested a beautiful experiment is indistinguishable from an artwork, and others questioned the notion of beauty existing at all in formulas. Indeed, it was an insightful and discerning exchange of ideas from five great thinkers at Stony Brook University.

“My theory is too beautiful not to be true.”

*Albert Einstein*

Valerio Jalongo is an Italian film director and screenwriter. Born in Rome, he attended the Gaumont School of Cinema, founded by producer Renzo Rossellini, and studied cinema at the University of Southern California in Los Angeles.

Sense of Beauty Panelists: Brooke Belisle, Assistant Professor, Art History and New Media; Robert P. Crease, Professor, Philosophy and History of Science; Sir Simon Donaldson, Professor, Fellow of the Royal Society, Mathematics, SCGP; Nobuho Nagasawa, Professor, Site-specific Installation, Interdisciplinary Practice, and Public Art; Peter van Nieuwenhuizen, Distinguished Professor, Physics.
UNIVERSITY GALLERY TOURS

Art brings everyone together, and in that spirit the biannual Stony Brook University Art Crawl opened its galleries across campus for the community at large. Organized and led by the gallery curators and students, free tours are offered each semester. Now in its fourth year, attendance was at a record high with many new and returning visitors joining the curatorial talks. When possible, the artists exhibiting their work are also present to answer questions. On October 24th, the fall tour commenced at the Wang Center and proceeded to Zuccaire Gallery, followed by the Melville Library Special Collections. The final destination was at the Simons Center Gallery, where the stroll was rounded out with a tea and cookie reception and an art full conversation to wrap up the day.

RESIDENCIES

Acclaimed photographers and filmmakers Anne Papillault and Jean-François Dars returned as Artists-in-Residence at the Simons Center in December 2018. Based in Paris, Papillault and Dars are Documentarians-in-Residence for the French National Center for Scientific Research since the 1980s. Once more, they took compelling portraits and photographs at the Center. An impromptu visit to IAS during their stay yielded excellent results as well. They continue to work on a special publication commemorating the ten-year anniversary of SCGP in November 2020, tentatively titled *The Spirit of the Place*. In this *poursuite de l’esprit*, Papillault and Dars unexpectedly captured some candid snapshots of a budding scientist at work. (See photo above.)

*Colorfolds*, Jenny Sabin, Digital rendering and drawing, 36 x 24 in, 2018
Image courtesy Jenny Sabin

*Sophie Komargodski at work*. Photo: Jean-François Dars

Art Crawl, Simons Center. Photo: Stony Brook University Libraries
2019-2020 Upcoming Events

Programs

Universality and Ergodicity in Quantum Many-body Systems: **August 26-October 18, 2019.** Organized by Boris Altshuler, Anatoly Dymarsky, Lea Santos, Jacobus Verbaarschot

Quantum-Mechanical Systems at Large Quantum Number: **August 26-September 20, 2019.** Organized by Luis Álvarez-Gaumé, Simeon Hellerman, Domenico Orlando, Susanne Reffert

Neural Networks and the Data Science Revolution: From Theoretical Physics to Neuroscience, and Back: **January 6-31, 2020.** Organized by Michael R. Douglas, Sergei Gukov, Jim Halverson, Sven Krippendorf, Fabian Ruehle, Giancarlo La Camera, Luca Mazzucato, Jin Wang

Renormalization and Universality in Conformal Geometry, Dynamics, Random Processes, and Field Theory: **February 3-June 5, 2020.** Organized by Kostya Khanin and Misha Lyubich

Geometrical Aspects of Topological Phases of Matter: Spatial Symmetries, Fractons and Beyond: **April 13-June 5, 2020.** Organized by Jennifer Cano, Dominic Else, Andrey Gromov, Siddharth Parameswaran, Yizhi You

Workshops

Simons Summer Workshop: **July 15-August 9, 2019.** Scientific organizers: Marilena Loverde, Cumrun Vafa. Local organizer: Martin Rocek

Graduate Summer School on the Geometry and Modular Representation Theory of Algebraic Groups: **August 19-23, 2019.** Organized by Mark Andrea de Cataldo, Francois Greer, Christian Schnell

Applications of Random Matrix Theory to Many-body Physics: **September 16-20, 2019.** Organized by Boris Altshuler, Anatoly Dymarsky, Lea Santos, Jacobus Verbaarschot

New Vistas on Vortices: **November 11-15, 2019.** Organized by Mathew Bullimore, Nuno M. Romão, Sushmita Venugopalan


Analysis, Dynamics, Geometry and Probability: **March 2-6, 2020.** Organized by Raanan Schul, Hrant Hakobyan, Kirill Lazebnik. Scientific committee: Peter Jones, Misha Lyubich, Dennis Sullivan

For the most up-to-date schedule, please visit scgp.stonybrook.edu/science

The SCGP welcomes proposals for scientific programs and workshops. To submit a proposal, please visit: scgp.stonybrook.edu/science/call-for-proposals

For possible sabbatical stays, please contact: Alexander Abanov at aabanov@scgp.stonybrook.edu
Dear Friends,

The Simons Center for Geometry and Physics is proud to share that we are launching a Friends of SCGP program.

The Friends of SCGP is a yearly giving program that provides donors with educational and cultural opportunities to engage with renowned scientists and to participate in the Center’s outreach activities.

As a supporter, you will enjoy an annual subscription to SCGP News, advanced notice of the Center’s public activities, VIP seating at special events, private invitations to exclusive chalk talks and more.

We have chosen the five Platonic solids to represent various levels within the Friends of SCGP program. A Platonic solid is a “most ideal” polyhedron, so that all of its faces are identical regular polygons (such as squares, equilateral triangles, etc.), and the number of faces meeting at each vertex is the same for every vertex. These solids are named after the Greek philosopher Plato, who believed them to be the building blocks of our Universe. There are only five such solids, with the first geometric proof of this fact known already to Euclid.

The benefit levels are organized into societies named after legendary Greek mountains: Athos, Parnassus and Olympus. Mount Athos, according to Greek legend, is a rock which one of Gigantes Athos threw against the Greek god Poseidon. Mount Athos is often referred to as the “Holy Mountain,” and is currently home to 20 Eastern Orthodox monasteries. According to Greek myths, the Mount Parnassus was home to Greek muses and was sacred to the Greek gods Dionysus and Apollo. As a home to muses, Parnassus became a center or poetry, music and learning in general. Mount Olympus is the highest mountain in Greece. According to Greek myths it is the home of Greek gods and the location of the Throne of Zeus.

It is not accidental that we have chosen Greek themes for the Friends of SCGP program. Ancient Greeks contributed greatly to mathematics and believed that mathematics and, in particular, geometry were absolutely essential in understanding the Universe.

Fostering the dissemination of high quality scientific knowledge is an important part of the Center’s mission and with your support we can be even more successful.

We thank you,

Luis Álvaro-Gaumé
Director, Simons Center for Geometry and Physics
Stony Brook University

TO LEARN MORE ABOUT THE FRIENDS OF SCGP PROGRAM PLEASE VISIT OUR WEBSITE: SCGP.STONYBROOK.EDU/FRIENDS-OF-THE-SCGP

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With your gift, you will become a member of the Athos society and receive a personalized certificate of gratitude, recognition in our biannual newsletter, a mailed copy of SCGP News, special advanced notice of upcoming public events, and invitations to exclusive Friends of the SCGP chalk talks.

PARNASSUS SOCIETY

The Parnassus society is reserved for friends who support the SCGP with a gift of $5,000 or more. Members will receive all of the benefits of the Athos society, plus reserved VIP seating at Simons Center public events, invitations to private dinners with scientists following special outreach events, and invitations to our exclusive Ask a Scientist events.

OLYMPUS SOCIETY

Become a member of the Olympus society and make a significant impact by donating $15,000 or more. Members will receive all of the benefits of the Parnassus society and Athos society as well as access to exclusive Director’s events and more.
Simons Center for Geometry and Physics, Stony Brook University, NY

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