Non-equilibrium Dynamics and Broken Symmetry

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Strongly Coupled Systems Away From Equilibrium
SCGP
motivation: non-equilibrium dynamics

• The challenge: how to characterize quantum **dynamics** far from equilibrium?

• Lack of broadly applicable **principles** and **techniques**

• Progress has been achieved in specific circumstances & models [see N. Andrei’s & J. Schmalian’s talks]
  - e.g. quench in BCS: Barankov, Levitov & Spivak PRL 93 (2004)
  - …

• **Holography** provides non-integrable, yet solvable examples
  - YM isotropization Chesler & Yaffe PRL 102 (2009)
  - CFT thermalization: de Boer et al. PRL 106 (2011)
  - …
approach: holography

Use holography to model strongly-correlated systems & dynamics

Solve bulk Einstein (+ matter) for given initial & boundary conditions

initial state, $H$  

deformations (quench)

1. rapid quench of superfluids: universal (?) noneq. phase transition

2. thermal quench of superconducting ring: KZ scaling
holography

Duality between Anti-de Sitter gravity and conformal field theory

QFT in D dimensions & its entire RG flow

≈

theory of gravity in D+1 dimensions

Geometry in majority (so far) of cases is AdS

Recent renewed progress at a direct construction via entanglement renormalisation and/or exact RG
learning from this duality

Quantum theory of gravity in general case has to take into account quantum effects: hard!

Classical theory of gravity

\[ \simeq \]

Strongly coupled large-N quantum field theory in D (flat) dimensions

we know precise microscopic origin only in a few cases (N=4 SYM, ABJM,...)

also known as `AdS/CFT’, but more general than CFT and AdS
holographic superfluid/conductor

Here specific model. Results more general (beyond holography?)

Holographic superfluid/conductor:

\[
S = \int d^{d+1}x \sqrt{-g} \left[ R + \frac{d(d-1)}{\ell^2} - \frac{1}{4} F^2 - |D\psi|^2 - m^2 |\psi|^2 \right]
\]

Complex scalar $\psi$ is dual to order parameter $O_\psi$

1) leading near-boundary behavior of $\psi$: source, subleading: expectation value

2) boundary conditions on A: D for superfluid, N for superconductor

3) String/M-theory embeddings exist
equilibrium

asymptotics: \[ \psi = \psi^{(1)} z^{d-\Delta} + \psi^{(2)} z^\Delta + \ldots \]

- finite chemical potential
- fixed \( \psi^{(1)} = 0 \)
- broken branch has \textit{lower} free energy
- \textit{2nd order} phase transition

T=0 limit of broken phase

\[ \sqrt{|O_\psi|/\mu} \]
Near equilibrium

Near-equilibrium retarded two-point function takes form

$$G_R(\omega, k) = \frac{Z(\omega, k)}{i\omega + k^2 + \frac{1}{\xi^2}}$$

For small $\omega,k$: linear in $\omega$, quadratic in $k \rightarrow z=2$

c & $\xi$ determined by in-falling quadratic fluctuations (\`QNM\')

$$\xi = \xi_0 \left| 1 - \frac{T}{T_c} \right|^{-\nu}$$

explicit solution near $T_c$: $\nu = 1/2$ (in all cases considered here)
Quantum Quench

Nonequilibrium dynamics of holographic superfluid

[Bhaseen, Gauntlett, JS, Simons, Wiseman, PRL 110 (2013)]
setup

Quench: sudden change of micro parameter
here: use source of order parameter

d=3 holographic superfluid: AdS$_4$ bulk

Einstein-Maxwell (D) + relevant scalar (dimension two)

homogeneous in boundary directions

→ solve initial value problem for (1+1) non-linear PDE
quantum quench

\[ J_\psi(t) = \delta e^{-\left(\frac{t}{\bar{\tau}}\right)^2} \]
the resulting dynamics

• The dynamics of this quench give rise to three distinct regimes

I. Oscillation

II. Decay to finite gap

III. Decay to zero gap

The behaviour shown in Fig. 2 is reminiscent of the dynamical phase diagram for a BCS superconductor [16], despite the fact that the holographic superfluid is strongly coupled, and that the effects of thermal damping are incorporated. Indeed, the persistent oscillations of the integrable BCS Hamiltonian are replaced here by an under-damped approach towards \(|\langle h_O(t)\rangle| = 0\), whilst the power-law damped BCS oscillations are replaced by an exponentially damped approach. The transition at \(\phi\) provides a finite temperature and collision dominated analogue of the collisionless Landau damping transition [16].

Emergent Temperature Scale.

We can gain further insight by considering the phase diagram as a function of the final temperature, \(T_f\), corresponding to the equilibrium temperature of the final state black hole. In Fig. 3(a) we plot \(T_f\) versus the parameter \(\phi\), showing that stronger quenches lead to greater final temperatures, consistent with the notion that the quench leads to heating. Using this relationship we may re-plot the data in Fig. 2 as a function of \(T_f\); see Fig. 3(b). The data collapse on to the equilibrium phase diagram of the holographic superfluid [35], as indicated by the solid line. The transition from II to III is associated with increasing \(T_f\) above \(T_c\). However, Fig. 3(b) contains more information than the equilibrium phase diagram; there is an emergent dynamical temperature scale \(T_{\phi}\), associated with \(\phi\), where the time dynamical phase transition
BCS case [Barankov, Levitov (2006)]

- The dynamics of this quench give rise to **three** distinct regimes

  I. Oscillation
  
  II. Decay to finite gap
  
  III. Decay to zero gap

- exhibit analogous phenomena in a strongly-coupled system, with thermal and collisional damping
  → **generic** mechanism within dynamical symmetry breaking leading to this behavior
region I

- dominant QNMs have

\[ \text{Re}(\omega) \neq 0 \quad \text{Im}(\omega) \neq 0 \]

- now \( \langle O_f \rangle \neq 0 \) so

\[
|\langle O(t) \rangle|^2 = |\langle O(t) \rangle + ce^{-i\omega t}|^2 \\
= |\langle O_f \rangle|^2 + |c|^2 e^{2\text{Im}(\omega)t} \\
+ 2e^{\text{Im}(\omega)t} \left( \text{Re} [\langle O_f c^* \rangle \cos(\text{Re}(\omega)t) \right) \\
- \text{Im} [\langle O_f c^* \rangle \sin(\text{Re}(\omega)t)]
\]
region II

- dominant QNMs have
  \[ \text{Re}(\omega) = 0 \quad \text{Im}(\omega) \neq 0 \]

- now \( \langle \mathcal{O}_f \rangle \neq 0 \) so

\[
|\langle \mathcal{O}(t) \rangle|^2 = |\langle \mathcal{O}(t) \rangle + ce^{-i\omega t}|^2 \\
= |\langle \mathcal{O}_f \rangle|^2 + |c|^2 e^{2\text{Im}(\omega)t} \\
+ 2|c|e^{\text{Im}(\omega)t}\text{Re}[\langle \mathcal{O}_f \rangle c^*]
\]
region III

- dominant QNMs have

\[
\text{Re}(\omega) \neq 0 \quad \text{Im}(\omega) \neq 0
\]

- but \( \langle \mathcal{O}_f \rangle = 0 \) so

\[
|\langle \mathcal{O}(t) \rangle| = |\langle \mathcal{O}(t) \rangle + c e^{-i\omega t}| = |c| e^{\text{Im}(\omega) t}
\]
this behavior is universal

1. S.c. phase transition: coalescence of two poles at $T_c$ at $\omega = 0$

2. Broken U(1) $\Rightarrow$ Single pole (i.e. mode) at $\omega = 0$ (Goldstone mode)

3. At $T=0$ leading poles are oscillatory in nature

$1 + 2 + 3 = \text{dynamical phase diagram!}$
Thermal Quench

winding number scaling of superconducting ring

[W. Zurek, JS, A. del Campo]
Quench: change temperature of bath or equivalently dim-less T x C

d=2 holographic superconducting ring: AdS$^3$ bulk

Maxwell (N) + marginal scalar (dimension two)

spatial dependence, periodic direction: $\Phi \sim \Phi + C$

→ solve initial value problem for (2+1) non-linear PDE
further details

Large N → classical gravity: what breaks the symmetry?

Must add noise: Hawking radiation from black hole (1/N effect)

Simple approach: sample boundary conditions from a Wiener process

\[
\langle \text{Re}\psi_s(t, \phi) \text{Re}\psi_s(t', \phi') \rangle = \alpha \delta(t - t') \delta(\phi - \phi')
\]

\[
\langle \text{Im}\psi_s(t, \phi) \text{Im}\psi_s(t', \phi') \rangle = \alpha \delta(t - t') \delta(\phi - \phi')
\]

parameter \(\alpha\): scaling does not depend, prefactors do (weakly)
example: generating $W = 2$
universality in thermal quenches

temperature quench through phase transition $\rightarrow$ KZ defect formation

fluctuation chooses phase randomly on region of size $\xi$

independent regions start to grow and join

a winding number of two is imprinted

temperature decreases $\sim \tau_Q$
winding number statistics: KZ scaling

average over noise realizations

\[ \langle |W| \rangle \sim \tau_Q^{-0.13 \pm 0.02} \]

from KZM:

\[ \langle |W| \rangle \sim \tau_Q^{-0.125} \]

KZ scaling of defects as a result of quench

[see also Das’ talk yesterday]
Conclusions
Conclusions

very interesting far-from-equilibrium physics is accessible at the intersection of holography and numerical relativity

covered two scenarios:

1) rapid quenches of superfluid
2) defect formation in superconducting ring

quantum quench in superfluid: nonequilibrium phase transition, universal mechanism → beyond holography?

KZM: results consistent with mean field. Other cases? 1/N corrections?