Particle-hole symmetry and the nature of the composite fermion

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Plan

• Composite fermions in FQHE
• Problem of particle-hole symmetry
• Composite fermions as Dirac fermions
• Consequences

Ref: DTS, PRX 5, 301027 (2015)
\[ \nu = \frac{n}{2n + 1} \]

\[ \nu = \frac{n + 1}{2n + 1} \]

(Jain's sequences)
Jain’s sequence of plateaux

- Using the composite fermion most observed fractions can be explained

Electrons

\[ \nu = \frac{n}{2n + 1} \]

Composite fermions

\[ \nu_{\text{CF}} = n \]

\[ \nu = \frac{n + 1}{2n + 1} \]

\[ \nu_{\text{CF}} = n + 1 \]
Composite fermions

- Flux attachment: statistics does not change by attaching an even number of flux quanta

\[ e = CF \]

- Map FQH states of electrons to IQH of composite fermions
HLR field theory

\[ \mathcal{L} = i \psi^\dagger (\partial_0 - i A_0 + i a_0) \psi - \frac{1}{2m} |(\partial_i - i A_i + i a_i) \psi|^2 + \frac{1}{2} \frac{1}{4\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \]

\[ b = \nabla \times a = 2 \times 2\pi \psi^\dagger \psi \quad \text{“flux attachment”} \]

mean field: \[ B_{\text{eff}} = B - b = B - 4\pi n \]

\[ \nu_{\text{CF}}^{-1} = \nu^{-1} - 2 \]

Halperin, Lee, Read 1993
HLR field theory

\[ \mathcal{L} = i \psi^\dagger (\partial_0 - iA_0 + ia_0) \psi - \frac{1}{2m} |(\partial_i - iA_i + ia_i) \psi|^2 + \frac{1}{2} \frac{1}{4\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \]

\[ b = \nabla \times a = 2 \times 2\pi \psi^\dagger \psi \]

Two main features:

- number of CFs = number of electrons
- Chern-Simons action for gauge field \( a \)

Both comes with the flux attachment procedure

Halperin, Lee, Read 1993
• CF explains the Jain’s sequences

Electrons

\[
\nu = \frac{n}{2n + 1}
\]

Composite fermions

\[
\nu_{\text{CF}} = n
\]

\[
\nu = \frac{n + 1}{2n + 1}
\]

\[
\nu_{\text{CF}} = n + 1
\]

Predicts Fermi liquid nature of the \(\nu = \frac{1}{2}\) state.
Particle-hole symmetry

- The most serious drawback of the HLR theory is the lack of particle-hole symmetry
Particle-hole symmetry

\[ \Theta |\text{empty}\rangle = |\text{full}\rangle \]
\[ \Theta c_k^\dagger \Theta^{-1} = c_k \]
\[ \Theta i \Theta^{-1} = -i \]

\[ \nu \rightarrow 1 - \nu \]

\[ \nu = \frac{1}{2} \rightarrow \nu = \frac{1}{2} \]

exact symmetry the Hamiltonian on the LLL, when mixing of higher LLs negligible

PH symmetry in the CF theory

PH conjugate pairs of FQH states

\[ \nu = \frac{n}{2n + 1} \quad \nu = \frac{n + 1}{2n + 1} \]

\[ \nu = 1/3 \quad \nu = 2/3 \]
PH symmetry in the CF theory

PH conjugate pairs of FQH states

\[ \nu = \frac{n}{2n + 1} \hspace{1cm} \nu = \frac{n + 1}{2n + 1} \]

\[ \nu = \frac{2}{5} \hspace{1cm} \nu = \frac{3}{5} \]
PH symmetry in the CF theory

PH conjugate pairs of FQH states

\[ \nu = \frac{n}{2n + 1} \quad \text{and} \quad \nu = \frac{n + 1}{2n + 1} \]

\( \nu = 3/7 \)

\( \nu = 4/7 \)
PH symmetry in the CF theory

PH conjugate pairs of FQH states

\[ \nu = \frac{n}{2n + 1} \quad \text{and} \quad \nu = \frac{n + 1}{2n + 1} \]

\( \nu = \frac{3}{7} \quad \text{and} \quad \nu = \frac{4}{7} \)

CF picture does not respect PH symmetry
PH symmetry of CF Fermi liquid?
PH symmetry of CF Fermi liquid?
PH symmetry of CF Fermi liquid?
• Particle-hole asymmetry of the HLR theory has been noticed early on Kivelson et al 1997

• No conclusive resolution has emerged until recently

• Possibility: spontaneous particle-hole symmetry breaking Barkeshli, Murugan, Fisher 2015

• Considerable numerical and some experimental evidence: $nu=1/2$ state is particle-hole symmetric

• Resolution: Dirac composite fermion
Dirac CF proposal
Dirac CF proposal

DTS, PRX 5, 301027 (2015)
Wang, Senthil
Metlitski, Vishwanath
Geraedts et al., 1508.04140
Mross, Alicea, Motrunich, 1510.08455
...

...
• CFs interact through an non-Chern-Simons $U(1)$ gauge field

• Number of CFs $\neq$ number of electrons
The composite fermion is a Dirac fermion. Particle-hole symmetry acts as time reversal:

\[ k \rightarrow -k \]
\[ \psi \rightarrow i\sigma_2\psi \]

- CFs interact through an non-Chern-Simons U(1) gauge field.
- Number of CFs ≠ number of electrons.
First indication of Dirac nature of CFs

\[ \nu = \frac{n}{2n + 1} \quad \rightarrow \quad \nu_{\text{CF}} = n \]

\[ \nu = \frac{n + 1}{2n + 1} \quad \rightarrow \quad \nu_{\text{CF}} = n + 1 \]
First indication of Dirac nature of CFs

\[ \nu = \frac{n}{2n + 1} \]

\[ \nu = \frac{n + 1}{2n + 1} \]

\[ \nu_{CF} = n + \frac{1}{2} \]
First indication of Dirac nature of CFs

\[ \nu = \frac{n}{2n + 1} \]

\[ \nu = \frac{n + 1}{2n + 1} \]

\[ \nu_{CF} = n + \frac{1}{2} ? \]

CFs form an IQH state at half-integer filling factor: possible only for a Dirac fermion
Figure 4 | QHE for massless Dirac fermions. Hall conductivity $\sigma_{xy}$ and longitudinal resistivity $\rho_{xx}$ of graphene as a function of their concentration at $B = 14$ T and $T = 4$ K. $\sigma_{xy} = (4e^2/h)\nu$ is calculated from the measured $\rho_{xx}$. The figure illustrates the quantized Hall effect in graphene.

Novoselov et al 2005
$$(\text{Particle-hole})^2$$

$\Theta^2 = \pm 1$
\[ \Theta |\text{empty}\rangle = |\text{full}\rangle = c_1^\dagger c_2^\dagger \cdots c_M^\dagger |\text{empty}\rangle \]

\[ \Theta c_k^\dagger \Theta^{-1} = c_k \]

anti-unitary
\[ \Theta |\text{empty}\rangle = |\text{full}\rangle = c_1^\dagger c_2^\dagger \cdots c_M^\dagger |\text{empty}\rangle \]

\[ \Theta c_k^\dagger \Theta^{-1} = c_k \]

anti-unitary

\[ \Theta^2 = (-1)^{M(M-1)/2} \]
\[ \Theta |\text{empty}\rangle = |\text{full}\rangle = c_1^\dagger c_2^\dagger \cdots c_M^\dagger |\text{empty}\rangle \quad \Theta c_k^\dagger \Theta^{-1} = c_k \]

anti-unitary

\[ \Theta^2 = (-1)^{M(M-1)/2} \]

\[ M = 2N_{\text{CF}} \quad \Theta^2 = (-1)^{N_{\text{CF}}} \]
\[ \Theta |\text{empty}\rangle = |\text{full}\rangle = c_1^\dagger c_2^\dagger \cdots c_M^\dagger |\text{empty}\rangle \quad \Theta c_k^\dagger \Theta^{-1} = c_k \]

anti-unitary

\[ \Theta^2 = (-1)^{M(M-1)/2} \]

\[ M = 2N_{\text{CF}} \quad \Theta^2 = (-1)^{N_{\text{CF}}} \]

\[ \Theta \text{ acts as time reversal on the Dirac composite fermion} \]

\[ \psi \rightarrow (i\sigma_2)^2 \psi \rightarrow (i\sigma_2)^2 \psi = -\psi \]

Geraedts, Zaletel, Mong, Metlitski, Vishwanath, Motrunich; Levin, Son
Problem with flux attachment

• For an even number of orbitals on the LLL,
\[ \Theta^2 |\text{any}\rangle = (-1)^{M/2} |\text{any}\rangle \]

• independent of the number of electrons in |any\rangle

• Thus for a consistent \( \Theta^2 \) assignment \( N_{\text{CF}} = M/2 \)
  • in general \( N_{\text{CF}} \neq N_e \)

• This goes against the philosophy of flux attachment
Tentative picture

\[ \mathcal{L} = i \bar{\psi} \gamma^\mu (\partial_\mu - ia_\mu) \psi - \frac{1}{4\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda + \frac{1}{8\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + \cdots \]

- Instead of flux attachment, the CF appears through particle-vortex duality
- Well known duality of bosons, new for fermions
- \( \psi \) is a Dirac fermion, no \( \text{ada} \)
- Jain sequences not affected
Particle-vortex duality

original fermion

magnetic field
density

composite fermion
density
magnetic field

\[ S = \int d^3x \left[ i\bar{\psi} \gamma^\mu (\partial_\mu + 2ia_\mu)\psi + \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda + \cdots \right] \]

\[ j^\mu = \frac{\delta S}{\delta A_\mu} = \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda \]

\[ \frac{\delta S}{\delta a_0} = 0 \rightarrow \langle \psi \gamma^0 \bar{\psi} \rangle = \frac{B}{4\pi} \]
Consequences of Dirac CF

If \( \hat{O} \) is particle-hole symmetric
i.e., \( \hat{O} = (\rho - \rho_0) \nabla^2 \rho \)

\[ \langle -k|\hat{O}|k \rangle = 0 \]
related to pi Berry phase

Suppression of Friedel oscillations

Geraedts et al, 2015
Some predictions

\[ j = \sigma_{xx} E + \sigma_{xy} E \times \hat{z} + \alpha_{xx} \nabla T + \alpha_{xy} \nabla T \times \hat{z} \]

- At exact half filling

\[ \sigma_{xy} = \frac{e^2}{2h} \quad \alpha_{xx} = 0 \]

distinct from HLR theory

Potter, Serbyn, Vishwanath 2015
Consequences

• New particle-hole symmetric gapped nonabelian state at $\nu=1/2$:

$$\langle \epsilon^{\alpha\beta} \psi_\alpha \psi_\beta \rangle \neq 0$$

• Particle-hole symmetry Pfaffian (PH-Pfaffian), analog of T-Pfaffian of interacting TIs

Pfaffian and anti-Pfaffian states: pairing of Dirac CFs with orbital angular momentum 2 and -2
But should flux attachment be exact?

• Flux attachment is sometimes described as an procedure that starts from an exact mapping of wavefunctions

\[ \psi(x_1, x_2, \ldots x_n) \rightarrow \prod_{ij} \frac{z_i - z_j}{|z_i - z_j|} \psi(x_1, x_2, \ldots x_n) \]

• This step is not yet useful. Additional arguments leading to the low-energy effective theory of the composite fermion are not exact.
Is it Dirac?

• Is there a Dirac cone at $p=0$? No, outside the scope of the low-energy effective theory (near Fermi surface).

• Two features:
  • zero local Berry curvature
  • $\pi$ Berry phase around the Fermi surface

• These are characteristics of interacting Landau Fermi liquids of massless Dirac fermions
Conclusion and open questions

• PH symmetry: a challenge for old CF picture
• Proposal: Dirac CF with gauge, non-CS interaction
• a fermionic particle-vortex duality

Open questions:
• Deriving the duality
• experimental measurement of the Berry phase
• PH-symmetric Pfaffian state?