Magnetotransport in Weyl and Dirac Metals

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Outline

- Introduction
- Berry Phase in Crystals
- Chiral Kinetic Theory
- Results and Open Questions
- Future Directions
Experimental realization of the first Dirac semimetal \( \text{Cd}_3\text{As}_2 \) in 2013.

Others were discovered in 2014 and 2015: \( \text{Na}_3\text{Bi}, \text{ZrTe}_5 \)…

Type-I Weyl semimetals were realized in 2015: \( \text{TaAs}, \text{NbAs}, \text{NbP}, \text{TaP} \).

Other gapless phases: type-II Weyl semimetals, quadratic band-touching points, loop semiconductors, "new fermions"…
Introduction

Neupane, M. et al. (2014)

ZrTe₅ - Li, Q. et al. (2014)

Na₃Bi - Liu, Z. K. et al. (2014)
Introduction

- ARPES experiments showed the presence of band-touching points.
- Quasiparticles are eigenstates of the Weyl Hamiltonian.
- What to expect?
- Chiral zero-modes in the presence of magnetic field.
Introduction

Chiral magnetic effect (CME).

Son and Spivak (2013)
Introduction

Negative magnetoresistance (MR) in the longitudinal direction.

\( \text{ZrTe}_5 \) - Li, Q. et al. (2014)

\( \text{Na}_3\text{Bi} \) - Xiong, J. et al. (2015)
Introduction

- What about Cd$_3$As$_2$?
- No conclusive signature of CME.
- Observation of Shubnikov-de Haas (SdH) effect: quantum oscillations in MR.
- Transitions between discrete Landau levels (LLs).
- “Large” Fermi surface: many filled LLs.

Cd$_3$As$_2$ - Ong, P.N. et al. (2014)
Introduction

- SdH effect: **metallic behavior**.
- Kinetic equations to study transport in Dirac/Weyl metals.
- Regime of interest: when **CME** and **quantum oscillations** are both pronounced.
- Known effects, not considered together.
- Can be potentially observed in cleaner samples of Cd$_3$As$_2$.
- Potentially relevant for ZrTe$_5$: unstable Dirac semimetal, no Lifshitz point.
Introduction

Goal: Study the interplay between CME and SdH oscillations.

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Berry Phase in Crystals

- Usual assumption: only quasiparticle states near the Fermi surface contribute to transport.

- How to know whether there was a Weyl point hidden inside a Fermi surface before turning on interactions?

- Quasiparticle states will pick up a phase when adiabatically moved around Fermi surface.
Berry Phase in Crystals

- Berry flux is zero for ordinary metals.
- **Weyl points** are monopoles solutions of the Berry curvature.

Berry flux is half of the solid angle inscribed by the loop.
Berry Phase in Crystals

**Nielsen-Ninomiya theorem:** Total Chern number across all pieces of Fermi surface must vanish. **Weyl points must come in pairs.**

**Dirac semimetals:** Quasiparticles belong to a Kramer’s doublet. Dirac nodes with a pair of Weyl points with opposite chirality.

**Weyl semimetals:** Weyl points separated in the Brillouin zone. **Time-reversal (inversion) symmetry is broken.**
Berry Phase in Crystals

Berry phase effects in quasiparticle dynamics: Sundaram and Niu 1999.

\[ \dot{x} = \frac{\nabla_k \varepsilon}{\hbar} - \dot{k} \times \Omega \]

\[ \hbar \dot{k} = -eE - e \dot{x} \times B \]

Anomalous velocity: Karplus and Luttinger (1954).

Berry curvature:

\[ \Omega(k) = i \nabla_k \times \langle u_k | \nabla_k u_k \rangle. \]
Berry Phase in Crystals

- New terms in the electric current:
  \[
  j = -\frac{e}{\hbar} \int_{BZ} \frac{d^3 k}{(2\pi)^3} f(k) \left[ \nabla_k \varepsilon + \frac{e}{\hbar} (\Omega \cdot \nabla_k \varepsilon) B + e E \times \Omega \right]
  \]

- CME and anomalous Hall effect (AHE).
- Distribution function obtained from Boltzmann equation.
- Linear response theory: conductivity tensor.
Outline

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- ✔ Berry Phase in Crystals
- □ Chiral Kinetic Theory
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Chiral Kinetic Theory

Distribution function in linear response:

\[ f(k) = f_0(\varepsilon) + e f'_0(\varepsilon) g(k) \cdot E \]

Equilibrium term is the Fermi-Dirac distribution.

Conductivity tensor:

\[
\sigma_{ab} = -\frac{e^2}{\hbar} \int \frac{\partial f_0}{\partial \varepsilon} g_b(k) \left( \nabla_k \varepsilon + \frac{e}{\hbar} (\nabla_k \varepsilon \cdot \Omega) B \right)_a \frac{d^3k}{(2\pi)^3}
\]

\[ + \frac{e^2}{\hbar} \epsilon_{abc} \int \Omega_c(k) f_0(\varepsilon) \frac{d^3k}{(2\pi)^3}. \]
Chiral Kinetic Theory

Underlying assumptions:

- Low temperature.

\[ \varepsilon_F \gg k_B T \]

- Dilute and homogenous impurity density. Single-impurity scattering approximation.

- Spherical Fermi surface (isotropy) and many LLs filled.

\[ \frac{1}{2} k_F^2 \ell_B^2 \gg 1, \quad \ell_B^2 = \frac{\hbar}{eB}. \]
Chiral Kinetic Theory

Conditions to treat each chirality independently:

- **Weyl metals**: Weyl points well-separated in the Brillouin zone. Large momentum transfer between inter-chirality scattering.

- **Dirac metals**: Inter-chirality scattering flips quasiparticle spin, $\mathbb{Z}_2$-breaking terms. Band mixing happens at much higher momenta.
Chiral Kinetic Theory

Case of interest: $\mathbf{B} = B\hat{z}$ and $\mathbf{E} = E\hat{z}$.

Seek for stationary solutions of Boltzmann equation.

Not possible for DC fields without chirality relaxation mechanism.

AC conductivity:

$$\omega, \frac{1}{\tau} \gg \frac{1}{\tau_v} \approx 0.$$ 

Inter-chirality scattering rate: frequency cutoff.
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Results and Open Questions

- Discreteness of LLs introduced in the conductivity formula via **Bohr-Sommerfeld quantization condition.**

\[
\int_{\varepsilon = \varepsilon_F} \frac{1}{2} \left( 1 + \frac{e}{\hbar} \mathbf{\Omega} \cdot \mathbf{B} \right) \hat{z} \cdot \mathbf{k} \times dk = \frac{2\pi n}{\ell_B^2}
\]

- Spherical Fermi surface:

\[
n = \frac{k_{\perp}^2 \ell_B^2}{2} - \frac{c_1(S_F)k_z}{\sqrt{k_{\perp}^2 + k_z^2}}
\]
Results and Open Questions

Expression for DC conductivity:

\[
\frac{\sigma_{zz}(B)}{\sigma_0} = 1 + 3 \left( \frac{\tau_v}{\tau} - \frac{4}{5} \right) \left( \frac{eB}{2\hbar k_F^2} \right)^2 + \frac{3}{\pi} \\
\times \sum_{l=1}^{\infty} e^{-\lambda_p l} \frac{\chi l}{\sinh \chi l} \left( \frac{eB}{\hbar k_F^2} \right)^{3/2} \cos \left( \frac{\pi l \hbar k_F^2}{eB} + \frac{\pi}{4} \right)
\]

Conductivity ratio to exclude non-universal temperature dependence.
Results and Open Questions

- Pronounced oscillations:
  - Temperature is much smaller than gap between LLs.
    \[ \lambda = 2\pi^2 k_B T \frac{k_F \ell_B^2}{\hbar v_F} \ll 1 \]
  - Coherence: many cyclotron orbits before colliding.
    \[ \lambda_D = 2\pi^2 \frac{k_F \ell_B^2}{v_F \tau_Q} \ll 1 \]
Results and Open Questions

Experimental parameters from Ong P.N. et al. (2014).

\[
\frac{\rho_{zz}}{\rho_0}
\]

\(\tau_Q = 0\)
\(\tau_Q = 8 \times 10^{-13} \text{s}\)
\(\tau_Q = 5 \times 10^{-14} \text{s}\)

Results and Open Questions

- Positive MR for weak magnetic field.

\[ \text{Cd}_3\text{As}_2 \] - Ong, P.N. \textit{et al.} (2014)  

- Non-analytical terms in the conductivity.
Results and Open Questions

- **Too strong to be weak anti-localization.**

- Not yet understood. Possible explanations:
  - Impurity screening highly dependent on magnetic field.
    
    \[ \tau = \tau(B) \]

  - Cannot treat each chirality independently.

  - Sample inhomogeneity (classical effect).
Results and Open Questions

However…


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Future Directions

Fermi arcs were observed in ARPES experiments.

TaAs - Xu, S. et al. (2015)
Future Directions

- Surface modes in SdH effect for Cd₃As₂.

Can this be captured by the chiral kinetic theory with boundary states?

Thank you!

And let us enjoy the cookies outside…