Nematic Order and Geometry in Fractional Quantum Hall Fluids

Eduardo Fradkin
Department of Physics and Institute for Condensed Matter Theory
University of Illinois, Urbana, Illinois, USA

Seminar at the Simons Center for Geometry and Physics
Stony Brook University
Stony Brook, New York, May 16, 2016

The Fractional Quantum Hall Effect

Energy levels and angular momentum transitions for a 2 DEG Al As - Ga As heterostructure.
Phases of the 2DEG in magnetic fields

- Fractional quantum Hall fluids are preeminent at high fields (or high densities) in Landau levels N=0,1

- On higher, N ≥ 2, Landau levels there are integer quantum Hall states

- At low densities Wigner crystals have been predicted (maybe seen)

- Compressible liquid crystal-like phases: nematic and stripe (‘bubble’) phases are seen for N ≥ 2 and in N=1 in tilted magnetic fields

- Compressible nematic phases do not exhibit the fractional (or integer) Hall effect

- Nematic phases: uniform fluids that break rotational invariance spontaneously

- Stripe phases: break translation and rotational invariance spontaneously

- Crystalline, nematic and stripe phases are fragile: couple strongly to disorder
Topological Quantum Hall Fluids

• Fractional quantum Hall fluids: uniform topological fluids with
  
  • a topologically protected Hall conductivity \( \sigma_{xy} = \nu \frac{e^2}{h} \), where \( \nu = \frac{N_e}{N_\phi} \) is the filling fraction of the Landau level

  • incompressible fluids with a finite energy gap

  • a ground state degeneracy \( m^g; m \in \mathbb{Z} \), \( g \) is the genus of the 2D surface

  • Excitations: `quasiparticles’ with fractional charge, fractional statistics (Abelian and non-Abelian), topological spin, and a quantum dimension

  • there are \( m \) distinct quasiparticle types (\( m = \) degeneracy on a torus)

  • have chiral edge states with universal properties

  • Entanglement entropy: \( S_{\text{VN}} = \text{const.} \ell - \ln \mathcal{D} + \ldots \) (with \( \mathcal{D}^2 = \sum_k d_k^2 \))
Electronic Nematic Fluids

- Uniform phase of a fluid that breaks spontaneously rotational invariance

- Order parameter: traceless symmetric tensor (in 2D this is equivalent to a director)

- In an electronic system a nematic state is a phase with a large, sharp temperature-dependent transport anisotropy (2DEG near $\nu=9/2$; Sr$_3$Ru$_2$O$_7$ for H~7-8T; YBCO in the pseudogap regime; BaFe$_2$As$_2$)

- Two pathways to an electronic nematic state: a) by melting a charge stripe phase (i.e. quantum or thermal melting a unidirectional CDW), and b) by a Pomeranchuk instability of a Fermi liquid: particle-hole condensate in the quadrupolar, $\ell=2$, channel)

$$Q(x) = \frac{1}{k_F^2} \begin{pmatrix} \psi^\dagger(x)(\partial_x^2 - \partial_y^2)\psi(x) & \psi^\dagger(x)2\partial_x\partial_y\psi(x) \\ \psi^\dagger(x)2\partial_x\partial_y\psi(x) & \psi^\dagger(x)(\partial_y^2 - \partial_x^2)\psi(x) \end{pmatrix}$$
Compressible Nematic Phases

J. Eisenstein et al, 1998

- Compressible nematic phases in the middle of Landau levels $N=2-6$
- For $N=1$ they appear in tilted fields replacing the non-Abelian FQH state
- Anisotropic longitudinal transport
- Nematic order parameter: $2\times2$ real traceless symmetric tensor $Q$ (quadrupole)
- $Q$ changes sign under a $90^\circ$ rotation

\[
Q = \begin{pmatrix} Q_{xx} & Q_{xy} \\ Q_{xy} & -Q_{xx} \end{pmatrix} \propto \begin{pmatrix} \rho_{xx} - \rho_{yy} & \rho_{xy} \\ \rho_{xy} & \rho_{yy} - \rho_{xx} \end{pmatrix}
\]

\[
Q = Q_{xx} + iQ_{yy}
\]
Close competition between nematic and paired FQH states at $\nu=5/2$

Observation of a transition from a topologically ordered to a spontaneously broken symmetry phase
Nature Physics 12, 191 (2016)
Evidence for a fractionally quantized Hall state with anisotropic longitudinal transport

Jing Xia\textsuperscript{1,*†}, J. P. Eisenstein\textsuperscript{1}, L. N. Pfeiffer\textsuperscript{2} and K. W. West\textsuperscript{2}

Nature Physics 7, 845 (2011)

This is a phase transition \textit{inside} a FQH state!
The Geometry of FQH fluids

- How does the topological fluid respond to changes in the geometry of the 2D surface?

- How is the concept of the Hall viscosity (i.e. a non-dissipative viscosity) related to a response to a change in the geometry?

- How is the nematic order related to changes in the actual geometry?

- We will now see that these concepts are actually related to each other and to the concept of flux attachment.
Flux Attachment and Geometry

- Flux attachment of \( k \) fluxes (\( k \) even) requires a topological spin

- If \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \) denote a local frame tangent to the worldline

- This frame couples to the actual spin connection of the 2D surface

- As a result, in addition to coupling to the metric of the 2D surface, the covariant derivative of the particles must be changed to

\[
D_\mu = \partial_\mu + i \left( A_\mu + \frac{k}{2} \omega_\mu \right)
\]
Full Effective Hydrodynamic Action for the \( \nu=1/(2k+1) \) Laughlin States

\[
\mathcal{L} = +\bar{\rho}\delta A_0 + \frac{2k + 1}{2} \bar{\rho}\omega_0 - \frac{2k + 1}{4\pi} \varepsilon^{\mu\nu\lambda} b_\mu \partial_\nu b_\lambda \\
- \frac{\varepsilon^{\mu\nu\lambda}}{2\pi} b_\mu \partial_\nu \delta A_\lambda - \frac{2k + 1}{2} \frac{\varepsilon^{\mu\nu\lambda}}{2\pi} b_\mu \partial_\nu \omega_\lambda - \frac{\varepsilon^{\mu\nu\lambda}}{48\pi} \omega_\mu \partial_\nu \omega_\lambda
\]

The Laughlin FQH state at \( \nu=1/(2k+1) \) is described by a \( \text{U}(1)_{2k+1} \) Chern-Simons theory

Hall viscosity

\[
\eta_H = \frac{\delta \mathcal{L}}{\delta \omega_0} = s \frac{\bar{\rho}}{2} = \frac{2k + 1}{2} \frac{\bar{\rho}}{2}
\]

The fractional spin \( s \) is the coefficient of the Wen-Zee term

The last term is the \textit{gravitational} Chern-Simons term for the abelian spin connection and controls the thermal transport (Stone 2012)

This result generalizes to all FQH states, abelian and non-abelian
Nematicity and Geometry

- The application of this framework to the nematic transition shows that
- The nematic fields mix with the frame fields of the background metric
- Upon integrating out the nematic fluctuations one finds the correct value of the Hall viscosity
- In the isotropic phase the Hall viscosity is universal
- In the nematic phase the Hall viscosity is modified if disclinations of the nematic order are present in the form of a quadrupolar tail
Theories of the Nematic FQH state


- The nematic fluctuations entered in the parity even corrections

- It required that the coefficient of the leading parity even term ("Maxwell") changes sign at the transition. But, this coefficient cannot be changed since it is given by the $k=0$ the gap of the Kohn (magneto-plasmon) mode, whose value is fixed to be the cyclotron energy by Galilean invariance.

- Maciejko, Hsu, Kivelson, Park and Sondhi (2013) proposed a phenomenological theory of the nematic FQH state

- They showed that the (quadrupolar) Girvin-MacDonald-Platzman mode condensation can drive the transition

- You, Cho and Fradkin (2014) provided microscopic derivation of the effective field theory and connected it with the theory of the Hall viscosity and the geometric response of FQH fluids.
Transition to a Nematic FQH State

- Rapid but continuous rise in the anisotropy in the transport at temperatures where the FQH state is already well formed.

- The transition does not close the gap of the FQH state

- It happens for a range of tilt angles

- Continuous phase transition to an incompressible nematic FQH state

- The transition is weakly rounded by the in-plane component of the magnetic field which acts as a rotation symmetry breaking field

- Possible mechanism: Condensation at $k=0$ of the Girvin-MacDonald-Platzman (GMP) collective mode (quadrupolar)

Spectrum of collective modes of the Laughlin $\nu=1/3$ FQH state

- Kohn mode (magnetoplasmon)
- GMP mode (magnetophonon)
- Magnetoroton
Fractional Quantum Hall Effect and Nematicity

\[ S = \int d^2x dt \left[ \Psi^\dagger(x) D_0 \Psi(x) - \frac{1}{2m_e} (D \Psi(x))^\dagger \cdot (D \Psi(x)) \right] \]
\[ - \frac{1}{2} \int d^2x' d^2x dt V(|x - x'|) (\rho(x) - \bar{\rho})(\rho(x') - \bar{\rho}) \]
\[ - \frac{1}{2} \int dt \int d^2x d^2x' F_2(|x - x'|) \text{Tr}[Q(x)Q(x')] \]

\[ F_2(q) = \frac{F_2}{1 + \kappa q^2} \quad \text{Quadrupolar Landau interaction of Fermi Liquids} \]

\[ D_\mu = \partial_\mu + i A_\mu \quad \text{Covariant derivative} \]

\[ Q(x) = \Psi^\dagger(x) \left( \begin{array}{cc} D_x^2 - D_y^2 & D_x D_y + D_y D_x \\ D_x D_y + D_y D_x & D_y^2 - D_x^2 \end{array} \right) \Psi(x) \]

Nematic order
class: parameter:

Quadrupolar fermionic density
Flux Attachment and Chern-Simons Gauge Theory

Each fermion is attached to $k \in \mathbb{Z}$ (even) fluxes maps fermions to fermions

$$S(\Psi^*, \Psi, A_\mu) \mapsto S(\Psi^*, \Psi, A_\mu + a_\mu) + \frac{1}{4\pi k} \int d^3 x \epsilon_{\mu \nu \lambda} a^\mu \partial^\nu a^\lambda$$

$$D_\mu = \partial_\mu + iA_\mu \mapsto D_\mu = \partial_\mu + i(A_\mu + a_\mu)$$

- Landau level at filling fraction $\nu = \frac{N_e}{N \phi}$
- The average flux is reduced to $N \phi^{\text{eff}} = N \phi - 2\pi k N e$: $\nu^{\text{eff}} = \frac{N_e}{N \phi^{\text{eff}}} = p \in \mathbb{Z}$
- “Composite fermions” filling up $p$ effective Landau levels (Jain, 1989)
- The allowed filling fractions are the Jain sequences: $\nu = p/(p k + 1)$
- Laughlin states: $p=1$ and $k=m-1 \Rightarrow \nu = 1/m$ ($m$ odd)
- At one-loop $\Rightarrow \sigma_{xy} = \nu e^2/h$, and maps the “composite fermions” into anyons with $e^* = e/m$, statistics: $\theta = \pi/m$; this is an incompressible topological fluid
- If $p \mapsto \infty$, $\nu \mapsto 1/k$; in this limit the gap vanishes and the system is a composite Fermi liquid

López and EF, 1991
Theory of the Nematic State for $\nu=1/3$

(trivial to extend to all Jain states)

$S = \int d^2x dt \left[ \Psi^\dagger(x) D_0 \Psi(x) - \frac{1}{2m_e} (D \Psi(x))^\dagger \cdot (D \Psi(x)) \right]$

\[ - \frac{1}{32\pi^2} \int d^2x' d^2x dt \ V(|\mathbf{x} - \mathbf{x}'|) \delta b(x) \delta b(x') \]

\[ + \frac{1}{8\pi} \int d^2x dt \ \epsilon^{\mu\nu\lambda} \delta a_\mu \partial_\nu \delta a_\lambda \]

\[ + \int d^2x dt \left[ \frac{1}{4F_2m_e^2} \mathbf{M}^2 + \frac{\kappa}{4F_2m_e^2} \sum_{i=1,2} |\nabla M_i|^2 \right] \]

\[ + \frac{M_1}{m_e} \Psi^\dagger (D_x^2 - D_y^2) \Psi + \frac{M_2}{m_e} \Psi^\dagger (D_x D_y + D_y D_x) \Psi \]

$\delta b = \epsilon_{ij} \partial_i a_j$

$M_1$ and $M_2$:
Hubbard-Stratonovich fields

$Q_{ij} = \begin{pmatrix} M_1 & M_2 \\ M_2 & -M_1 \end{pmatrix}$

- The 2x2 traceless symmetric tensor $Q$ couples to the composite fermions as a fluctuation of a two-dimensional metric tensor
- Nematic fluctuations are geometric degrees of freedom
Effective Action for the Nematic Fields

\[ \mathcal{L}_M = \frac{\epsilon^{ij} \bar{\rho}}{2} M_i \partial_0 M_j - r M^2 - \frac{\bar{\kappa}}{2} (\nabla M_i)^2 - \frac{u}{4} (M^2)^2 \]

\[ r = -\frac{1}{4F_2 m_e^2} - \frac{\bar{\omega}_c}{2\pi l_b^2}, \quad \bar{\kappa} = -\frac{\kappa}{2F_2 m_e^2} - \frac{5}{2\pi}, \quad u = \frac{b\bar{\omega}_c}{4\pi} \]

- First term: Berry phase that determines the commutation relations of the nematic fields
- This term requires that the dynamics has \( z=2 \)
- Its coefficient is the Hall viscosity of the composite fermions (not of the FQH fluid!)
- Nematic transition: \( r=0 \) and for \( F_2 = F_2^c < 0 \), \( |F_2^c| = \frac{\pi l_b^2}{2\bar{\omega}_c m_e^2} \)
- In the nematic phase \( \langle M \rangle \neq 0 \)
- A computation of the collective mode spectra shows that the GMP mode condenses at \( k=0 \) for \( r=0 \)
The nematic spin connection and disclinations

We can define a “spin connection” $\omega_\mu (Q)$ from the nematic fields (associated with local rotations) which couples to the gauge fields as a Wen-Zee term

\[
\begin{align*}
\omega_0 &= \epsilon^{ij} M_i \partial_0 M_j, \\
\omega_x &= \epsilon^{ij} M_i \partial_x M_j - (\partial_x M_2 - \partial_y M_1), \\
\omega_y &= \epsilon^{ij} M_i \partial_y M_j + (\partial_x M_1 + \partial_y M_2),
\end{align*}
\]

- The flux of the “spin connection” defines a curvature which is equal to a disclination of the nematic order!
- The disclination is the vorticity of the nematic order
- It carries fractional charge and fractional statistics

\[
\mathcal{L}_{wz} = \frac{1}{4\pi} \epsilon^{\mu\nu\rho} \omega_\mu \partial_\nu (\delta a_\rho + \delta A_\rho)
\]
The compressible nematic state

• Although similar ideas can, in principle, be applied to describe a compressible nematic state, there are problems

• In the compressible limit the gap vanishes and the flux attachment yields an uncontrollable amount of Landau level mixing

• There is no effective projection onto a Landau level

• There is no small expansion parameter to control the theory

• The fluctuations of a gauge field coupled to a Fermi surface lead to strong forward scattering infrared divergencies and to a breakdown of Fermi liquid theory: the quasiparticles are ill defined

• A naive computation of the Landau parameters of the CFL predicts that they are all equal to the Pomeranchuk instability value (in all channels!)

• Particle-hole symmetry is not possible

• Nevertheless we proceed and try to do our best
Effective Action of the Nematic Fields (one loop)

\[ S_{\text{eff}}[\mathbf{M}] = \int_{\mathbf{p}, \Omega} M_i(\mathbf{p}, \Omega) \mathcal{L}_{ij}(\mathbf{p}, \Omega) M_j(\mathbf{p}, \Omega) \]

\[ \mathcal{L}_{ij}(\mathbf{p}, \Omega) = \begin{pmatrix}
  -\frac{\kappa}{F_2} \mathbf{p}^2 - \delta - \cos^2(2\theta_\mathbf{p}) \frac{1}{2\pi \ell_0^3} \left( \frac{i\Omega}{|\mathbf{p}|} \right) & \sin(2\theta_\mathbf{p}) \cos(2\theta_\mathbf{p}) \frac{1}{2\pi \ell_0^3} \left( \frac{i\Omega}{|\mathbf{p}|} \right) \\
  \sin(2\theta_\mathbf{p}) \cos(2\theta_\mathbf{p}) \frac{1}{2\pi \ell_0^3} \left( \frac{i\Omega}{|\mathbf{p}|} \right) & -\frac{\kappa}{F_2} \mathbf{p}^2 - \delta - \sin^2(2\theta_\mathbf{p}) \frac{1}{2\pi \ell_0^3} \left( \frac{i\Omega}{|\mathbf{p}|} \right) 
\end{pmatrix} \]

- Nematic fluctuations are Landau damped (Oganesyan et al. 2001)
- Dynamic critical exponent \( z = 3 \)
- No Berry phase term at one loop order
- Breaking of time-reversal is in the Chern-Simons term of the gauge fields
- Effective (quadrupolar) couplings mix the nematic and the gauge field fluctuations
Beyond one-loop: Berry phase!

\[ S = \int_{p,\Omega} \chi \epsilon^{ij} \Omega M_i(p,\Omega) M_j(-p,-\Omega), \quad \chi \approx \frac{2}{3\pi \ell^2} \]

- Upon integrating-out the gauge field fluctuations we find a Berry phase term in the nematic action
- The coefficient \( \chi \) is not universal
- The Berry phase is subleading to the Landau damping terms
- At criticality and in the nematic phase it leads to a mixing of longitudinal (underdamped) and transverse (Landau damped) modes
- (non-universal) Wen-Zee term induced at two loop order
Nematic phase: transport anisotropy

- The conductivity can be computed from the current-current correlation function in the nematic state.

- The longitudinal conductivity $\sigma_{xx}$ is

$$\sigma_{xx} \sim \frac{\omega(1+M_1)/V(0)}{(1+M_1 \cos(2\theta_q))i\frac{\omega}{qV(0)i_0} + \frac{\omega^2}{V(0)^2q^2} - q^2/4}$$

- Deep in the nematic phase $M_1 \sim O(1)$ and the damping term in the denominator is very small for $\theta_q=\pi/2$.

- Resonance peak in the spectrum at 100 MHz ($\sim 4$ mK).

- Coupling to the weak lattice anisotropy gaps-out the overdamped Goldstone mode.

Sambandmurthy et al (PRB 2008)
Conclusions

• 2DEGs in large magnetic fields have a rich variety of phases

• The topological fractional quantum Hall fluids have, in addition to their universal topological properties, have a universal geometric response

• Nematic phases represent local quadrupolar fluctuations of the 2DEG that behave as a dynamical metric

• Nematic phases have universal properties when they coexist with incompressible fractional quantum Hall fluids

• Nematic phases also occur as compressible “metallic” phases

• Nematic order appears in close proximity of paired fractional quantum Hall states

• This suggests that these states have a common origin