Gauge theory for the cuprates near optimal doping

Developments in Quantum Field Theory and Condensed Matter Physics
Simons Center for Geometry and Physics,
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arXiv:1811.xxxxx
Fermi Surface and Pseudogap Evolution in a Cuprate Superconductor


Simultaneous Transitions in Cuprate Momentum-Space Topology and Electronic Symmetry Breaking

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507

**Figure 6**

Across the quantum critical point. 

a) Normal-state electronic specific heat in the $T=0$ limit as a function of doping, plotted as $C_{el}/T$ vs $p$ (red symbols) in Eu-LSCO (squares), Nd-LSCO (circles) and LSCO (diamonds). From ref. (75). We also show $C_{el}/T$ in YBCO (blue dots (18)) and in Tl2201 (green dot (76)). The vertical grey lines mark the limits of the CDW phase in Nd-LSCO, between $p=0.08$ and $p'=0.19$.

b) Normal-state Hall number $n_H$ in the $T=0$ limit as a function of doping, in YBCO (blue circles (21), $p'=0.19$) and Nd-LSCO (red squares (4), $p'=0.23$). We also show $n_H$ in LSCO (grey squares (67)) and YBCO (grey circles (68)) at low doping, and $n_H$ in Tl2201 (white diamond (29)) at high doping.

**5. PSEUDOGAP PHASE**

DOS: Density of states ($N_F$): Condensation energy $E_c$:

Upper critical field $H_{c2}$: Lower critical field $H_{c1}$: Residual linear term in the specific heat, $C(T)$ at $T=0$, purely electronic

The two traditional signatures of the pseudogap phase are: 1) a loss of density of states (DOS) below $p$; 2) the opening of a partial spectral gap below $T$, see by ARPES (Figs. 1c, 1d) and optical conductivity, for example. Here we summarize recent high-field measurements of the specific heat in the LSCO family (75) showing that there is a large mass enhancement at $p$. The new data show that the pseudogap does not simply cause a loss of DOS below $p$; instead, there is huge peak in the DOS at $p$ (Fig. 6a) – much larger than expected from a van Hove singularity (75, 80). We then show how high-field measurements of the Hall coefficient reveal a new signature of the pseudogap phase – a rapid drop in the carrier density, at $p$ (Fig. 6c). These new properties alter profoundly our view of the pseudogap phase, and of the strange metal just above it (sec. 6).
1. SU(2) gauge theory with adjoint scalars
   Confining and Higgs phases in 2+1 dimensions

2. SU(2) gauge theory of fluctuating antiferromagnetism

3. Role of fermions
   Pseudogap as a FL* phase
SU(2) gauge theory

SU(2) gauge theory with $N_h$ adjoint Higgs fields $H^a_\ell$ ($a = 1, 2, 3$, $\ell = 1 \ldots N_h$), with potential $V(H^a_\ell)$ and SU(2) gauge field $A^a_\mu$. There is $O(N_h)$ global flavor symmetry.

$$
\mathcal{L} = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} \left( \partial_\mu H^a_\ell - \epsilon_{abc} A^b_\mu H^c_\ell \right)^2 + V(H^a_\ell)
$$

$$
F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon_{abc} A^b_\mu A^c_\nu
$$

$$
V(H^a_\ell) = s H^a_\ell H^a_\ell + u_0 H^a_\ell H^a_\ell H^b_m H^b_m + u_1 \left( H^a_\ell H^a_m H^b_\ell H^b_m - \frac{1}{N_h} H^a_\ell H^a_\ell H^b_\ell H^b_m \right)
$$

\[ N_h = 1 \]

**Phase diagrams of SU(2) gauge theory**

- Condensation of \( H^a \) breaks SU(2) to U(1)
- U(1) confines because of proliferation of ’tHooft-Polyakov monopoles
- Monopole action \( \sim \sqrt{-s} \), leading to an exponentially large confinement scale
There is a global $O(N_h)$ symmetry, and we can define a gauge-invariant order parameter 

$$Q_{\ell m} = H^a_{\ell} H^a_m - \frac{\delta_{\ell m}}{N_h} H^a_n H^a_n$$

For $u_1 < 0$, the Higgs condensate is of the form

$$H^a_1 = (H_0, 0, 0), \quad H^a_2 = (0, 0, 0)$$

This breaks gauge $SU(2)$ to $U(1)$, and the $U(1)$ confines, as for $N_h = 1$. But the global $O(2)$ is broken because 

$$\langle Q_{\ell m} \rangle \neq 0$$
Phase diagrams of SU(2) gauge theory

- Condensation of $H^a$ breaks SU(2) to U(1), which confines

- Broken O(2) symmetry, $\langle Q_{\ell m} \rangle \neq 0$.
\[ N_h = 2 \]

Phase diagrams of SU(2) gauge theory

\[ V(H_\ell^a) \]

\[ \langle H_\ell^a \rangle \neq 0 \]

\[ V(H_\ell^a) \]

\[ \langle H_\ell^a \rangle = 0 \]

Higgs

\[ u_1 < 0 \]

- Condensation of \( H^a \) breaks SU(2) to U(1), which confines

- Broken O(2) symmetry, \( \langle Q_{\ell m} \rangle \neq 0 \).

Confinement

Possible Deconfined critical SU(2) gauge theory

Multi-universality of a Landau-allowed transition?

\[ S \]
$N_h = 2$

- There is a global $O(N_h)$ symmetry, and we can define a gauge-invariant order parameter

$$Q_{\ell m} = H_\ell^a H_m^a - \frac{\delta_{\ell m}}{N_h} H_n^a H_n^a$$

- For $u_1 < 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, 0, 0)$$

This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1$. But the global $O(2)$ is broken because

$$\langle Q_{\ell m} \rangle \neq 0$$

- For $u_1 > 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, H_0, 0)$$

This breaks gauge SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order (as in the ‘toric code’). But now the global $O(2)$ is unbroken because

$$\langle Q_{\ell m} \rangle = 0$$
\[ N_h = 2 \]

**Higgs**

\[ V(H^a) \]

\[ \langle H^a \rangle \neq 0 \]

\[ H^a \]

\[ u_1 > 0 \]

- Condensation of \( H^a \) breaks SU(2) to \( \mathbb{Z}_2 \), leading to \( \mathbb{Z}_2 \) topological order.

- Unbroken O(2) symmetry, \( \langle Q_{\ell m} \rangle = 0 \).

**Phase diagrams of SU(2) gauge theory**

\[ V(H^a) \]

\[ \langle H^a \rangle = 0 \]

\[ H^a \]

Possible Deconfined critical SU(2) gauge theory

Confinement
For $u_1 < 0$, the Higgs condensate is of the form

\[ H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, 0, 0) \quad , \quad H_3^a = (0, 0, 0) \]

This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1, 2$. The global O(3) is broken to O(2) and

\[ \langle Q_{\ell m} \rangle \neq 0 \]
Phase diagrams of SU(2) gauge theory

\[ N_h = 3 \]

\[ V(H^a_\ell) \]

\[ \langle H^a_\ell \rangle \neq 0 \]

\[ H^a_\ell \]

\[ \langle H^a_\ell \rangle = 0 \]

\[ H^a_\ell \]

\( u_1 < 0 \)

- Condensation of \( H^a \) breaks SU(2) to U(1), which confines

- O(3) symmetry broken to O(2), \( \langle Q_{\ell m} \rangle \neq 0 \).

Possible Deconfined critical SU(2) gauge theory

Multi-universality of a Landau-allowed transition?
• For $u_1 < 0$, the Higgs condensate is of the form

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$$\langle Q_{\ell m} \rangle \neq 0$$

• For $u_1 > 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0), \quad H_2^a = (0, H_0, 0), \quad H_3^a = (0, 0, H_0)$$

This breaks gauge SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order (as in the ‘toric code’). But now the global O(3) is unbroken because

$$\langle Q_{\ell m} \rangle = 0$$
Phase diagrams of SU(2) gauge theory

\[ N_h = 3 \]

\[ V(H^a_\ell) \]

\[ \langle H^a_\ell \rangle \neq 0 \]

\[ V(H^a_\ell) \]

\[ \langle H^a_\ell \rangle = 0 \]

**Higgs**

\[ u_1 > 0 \]

- Condensation of \( H^a \) breaks SU(2) to \( \mathbb{Z}_2 \), leading to \( \mathbb{Z}_2 \) topological order.

- Unbroken O(3) symmetry, \( \langle Q_{\ell m} \rangle = 0 \).

**Confinement**

Possible Deconfined critical SU(2) gauge theory
For \( u_1 < 0 \), the Higgs condensate is of the form

\[
H_1^\alpha = (H_0, 0, 0) \quad , \quad H_{\ell>1}^\alpha = (0, 0, 0)
\]

This breaks gauge SU(2) to U(1), and the U(1) confines, as for \( N_h = 1, 2 \). The global O(\( N_h \)) is broken to O(\( N_h - 1 \)) and

\[
\langle Q_{\ell m} \rangle \neq 0
\]
Phase diagrams of SU(2) gauge theory

\[ N_h \geq 4 \]

\[ V(H^a_\ell) \]

\[ \langle H^a_\ell \rangle \neq 0 \]

\[ u_1 < 0 \]

- Condensation of \( H^a \) breaks SU(2) to U(1), which confines

- \( O(N_h) \) symmetry broken to \( O(N_h - 1) \), \( \langle Q_{\ell m} \rangle \neq 0 \).

Confinement

Possible Deconfined critical SU(2) gauge theory

Multi-universality of a Landau-allowed transition?
\( N_h \geq 4 \)

- For \( u_1 < 0 \), the Higgs condensate is of the form

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\[\langle Q_{\ell m} \rangle \neq 0\]

- For \( u_1 > 0 \), the Higgs condensate is of the form

\[
H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, H_0, 0) \quad , \quad H_3^a = (0, 0, H_0) \quad , \quad H_{\ell > 3}^a = (0, 0, 0)
\]

This breaks gauge SU(2) to \( \mathbb{Z}_2 \), leading to \( \mathbb{Z}_2 \) topological order (as in the ‘toric code’). But now the global O\((N_h)\) is broken to O\((N_h - 3) \times \text{O}(3)\), and

\[\langle Q_{\ell m} \rangle \neq 0\]
Phase diagrams of SU(2) gauge theory

$N_h \geq 4$

\[ V(H^a \ell) \]
\[ \langle H^a \ell \rangle \neq 0 \]

\[ u_1 > 0 \]

- Condensation of $H^a$ breaks SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order.

- $O(N_h)$ symmetry broken to $O(N_h - 3) \times O(3)$, $\langle Q_{\ell m} \rangle \neq 0$.

Possible Deconfined critical SU(2) gauge theory

Confinement
1. SU(2) gauge theory with adjoint scalars
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Gauge theory of fluctuating antiferromagnetism

We can (exactly) transform the Hubbard model to the “spin-fermion” model:

**electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = -\sum_{i,\rho} t_{\rho} \left( c_{i,\alpha}^{\dagger} c_{i+\nu_{\rho},\alpha} + c_{i+\nu_{\rho},\alpha}^{\dagger} c_{i,\alpha} \right)$$

$$-\mu \sum_i c_{i,\alpha}^{\dagger} c_{i,\alpha} + \mathcal{H}_{\text{int}}$$

are coupled to a magnetic moment order parameter $\Phi^p(i), p = x, y, z$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \Phi^p(i) c_{i,\alpha}^{\dagger} \sigma_{\alpha\beta}^p c_{i,\beta} + V_{\Phi}$$
Gauge theory of fluctuating antiferromagnetism

For fluctuating antiferromagnetism (spin density waves (SDW)), we transform to a rotating reference frame using the SU(2) rotation $R_i$

$$
\begin{pmatrix}
  c_{i\uparrow} \\
  c_{i\downarrow}
\end{pmatrix} = R_i \begin{pmatrix}
  \psi_{i,+} \\
  \psi_{i,-}
\end{pmatrix},
$$

in terms of fermionic “chargons” $\psi_s$ and a Higgs field $H^a(i)$

$$
\sigma^p \Phi^p(i) = R_i \sigma^a H^a(i) R_i^\dagger
$$

The Higgs field is the SDW order in the rotating reference frame.
For (fluctuating) SDW SRO, we transform to a rotating reference frame using the SU(2) rotation

\[ R^i \],

in terms of fermionic "chargons" \( s \) and a Higgs field \( H^a(\tilde{x}) \).

The Higgs field is the SDW order in the rotating reference frame. Note that this representation is ambiguous up to a SU(2) gauge transformation, \( V^i \)

\[
\begin{pmatrix}
\psi_{i,+} \\
\psi_{i,-}
\end{pmatrix}
\rightarrow V_i \begin{pmatrix}
\psi_{i,+} \\
\psi_{i,-}
\end{pmatrix}
\]

\[ R_i \rightarrow R_i V^i \]

\[ \sigma^a H^a(\tilde{x}) \rightarrow V_i \sigma^b H^b(\tilde{x}) V_i^\dagger. \]

Note that this representation is ambiguous up to a SU(2) gauge transformation, \( V_i \)

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S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB 80, 155129 (2009)
### Table I. Quantum numbers of the matter fields in \( L \) and \( L_g \). The transformations under the SU(2)'s are labelled by the dimension of the SU(2) representation, while those under the electromagnetic U(1) are labeled by the U(1) charge. The antiferromagnetic spin correlations are characterized by (5.3). The Higgs field determines local spin correlations via (5.12). A summary of the charges carried by the fields in the resulting SU(2) gauge theory, \( L_g \), is in Table I. This rotating reference frame perspective was used in the early work by Shraiman and Siggia on lightly-doped antiferromagnets [89, 90], although their attention was restricted to phases with antiferromagnetic order. The importance of the gauge structure in phases without antiferromagnetic order was clarified in Ref. [85].

Given the SU(2) gauge invariance associated with (5.6), when we express \( L \) in terms of \( \sigma \) we naturally obtain a SU(2) gauge theory with an emergent gauge field \( A a \mu = (A a \sigma, A a \sigma) \), with \( a = 1, 2, 3 \).

We write the Lagrangian of the resulting gauge theory as [85–87]

\[
L_{g} = L_c + L_Y + L_R + L_H. \tag{5.9}
\]

The first term for the fermions descends directly from the \( L_c \) for the electrons \( X_i^{\dagger}s \beta \partial_\mu \sigma ss_0 + iA_a \sigma ss_0 + X_i,j^{\dagger}t_{ij}s, \tag{5.10} \) and uses the same hopping terms for \( c \) as those for \( c \), along with a minimal coupling to the SU(2) gauge field. Inserting (5.6) into \( L_{cn} \), we find that the resulting expression involves 2 complex Higgs fields, \( H_\alpha x \) and \( H_\alpha y \), which are SU(2) adjoints; these are defined by

\[
H_\alpha x = \frac{1}{\sqrt{2}} \text{Tr}[\sigma R_\alpha R^]\tag{5.11}
\]

and similarly for \( H_\alpha y \). Let us also note the inverse of (5.11)

\[
\sigma_\alpha^{-1} = \frac{1}{\sqrt{2}} \text{Tr}[\sigma R_\alpha R^]\tag{5.12}
\]

Begin with an effective theory for the Higgs field alone.
Gauge theory of fluctuating antiferromagnetism

We obtain different numbers of adjoint Higgs scalars, $N_h$, depending upon the spatial dependence of the local spin correlations:

**Neel correlations:** $N_h = 1$,
\[ \mathbf{K} = (\pi, \pi), \]
\[ H^a(i) = H^a_1(\mathbf{r}) e^{i\mathbf{K} \cdot \mathbf{r}_i} \]

**Canted antiferromagnetic correlations:** $N_h = 2$,
\[ \mathbf{K} = (\pi, \pi), \]
\[ H^a(i) = H^a_1(\mathbf{r}) e^{i\mathbf{K} \cdot \mathbf{r}_i} + H^a_2(\mathbf{r}) \]

**Unidirectional incommensurate correlations:** $N_h = 2$,
\[ \mathbf{K} = (\pi, \pi - \delta), \]
\[ H^a(i) = \text{Re} \left\{ [H^a_1(\mathbf{r}) + iH^a_2(\mathbf{r})] e^{i\mathbf{K} \cdot \mathbf{r}_i} \right\} \]

**Bidirectional incommensurate correlations:** $N_h = 4$,
\[ \mathbf{K}_y = (\pi, \pi - \delta), \mathbf{K}_x = (\pi - \delta, \pi), \]
\[ H^a(i) = \text{Re} \left\{ [H^a_1(\mathbf{r}) + iH^a_2(\mathbf{r})] e^{i\mathbf{K}_x \cdot \mathbf{r}_i} + [H^a_3(\mathbf{r}) + iH^a_4(\mathbf{r})] e^{i\mathbf{K}_y \cdot \mathbf{r}_i} \right\} \]
Gauge theory of fluctuating antiferromagnetism

For the hole-doped cuprates, \( N_h = 4 \), we define complex Higgs fields

\[
\mathcal{H}_x^a = H_1^a + iH_2^a, \quad \mathcal{H}_y^a = H_3^a + iH_4^a.
\]

The SU(2) gauge theory is

\[
\mathcal{L} = \frac{1}{2} |\partial_\mu \mathcal{H}_x^a - \epsilon_{abc} A_\mu^b \mathcal{H}_x^c|^2 + \frac{1}{2} |\partial_\mu \mathcal{H}_y^a - \epsilon_{abc} A_\mu^b \mathcal{H}_y^c|^2
\]

\[
+ \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + V(\mathcal{H}_x^a, \mathcal{H}_y^a)
\]

\[
F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c
\]

\[
V(\mathcal{H}_x^a, \mathcal{H}_y^a) = s \left( \mathcal{H}_x^a \mathcal{H}_x^a + \mathcal{H}_y^a \mathcal{H}_y^a \right) + u_0 \left( \mathcal{H}_x^a \mathcal{H}_x^a + \mathcal{H}_y^a \mathcal{H}_y^a \right)^2
\]

\[
+ \frac{u_1}{4} \left( \mathcal{H}_x^a \mathcal{H}_x^a - \mathcal{H}_y^a \mathcal{H}_y^a \right)^2 + \frac{u_2}{2} \left[ |\mathcal{H}_x^a \mathcal{H}_x^a|^2 + |\mathcal{H}_y^a \mathcal{H}_y^a|^2 \right]
\]

\[
+ u_3 \left( |\mathcal{H}_x^a \mathcal{H}_y^a|^2 + |\mathcal{H}_x^a \mathcal{H}_y^a|^2 \right).
\]
Gauge theory of fluctuating antiferromagnetism

There are multiple order parameters for different broken symmetries (Note: spin rotations are preserved and there is no SDW order)

- Ising nematic order
  \[ \phi = \mathcal{H}_x^a \mathcal{H}_x^a - \mathcal{H}_y^a \mathcal{H}_y^a \]

- Charge density wave (CDW) order at wavevectors \(2K_{x,y}\)
  \[ \Phi_x = \mathcal{H}_x^a \mathcal{H}_x^a, \quad \Phi_y = \mathcal{H}_y^a \mathcal{H}_y^a \]

- Charge density wave (CDW) order at wavevectors \(K_{x \pm K_y}\)
  \[ \Phi_+ = \mathcal{H}_x^a \mathcal{H}_y^a, \quad \Phi_- = \mathcal{H}_x^a \mathcal{H}_y^{a*} \]

- (Modulated) scalar spin chirality
  \[ \chi_{ijk} = \epsilon_{abc} \mathcal{H}_i^a(r_i) \mathcal{H}_j^b(r_j) \mathcal{H}_k^c(r_k) \]
Broken symmetries and topological order in the Higgs phase

\[ u_3 < 0 \]

\[ u_2 / |u_3| \]

\[ \begin{align*}
(C) & \quad \langle \phi \rangle \neq 0 \\
& \quad \mathbb{Z}_2 \\
(D) & \quad \langle \Phi_+ \rangle \neq 0 \\
& \quad \mathbb{Z}_2 \\
(B) & \quad \langle \Phi_x \rangle = \langle \Phi_y \rangle \neq 0 \\
& \quad \langle \Phi_+ \rangle = \langle \Phi_- \rangle \neq 0 \\
& \quad \text{U}(1) \\
(A) & \quad \langle \phi \rangle, \langle \Phi_x \rangle \neq 0 \\
& \quad \text{U}(1)
\end{align*} \]
Broken symmetries and topological order in the Higgs phase

\[ u_3 > 0 \]

(C) \( \langle \phi \rangle \neq 0 \)
\[ \mathbb{Z}_2 \]

(U(1)) \( \langle \phi \rangle, \langle \Phi_x \rangle \neq 0 \)

(F) \( \langle \Phi_\pm \rangle = \langle \Phi_\mp \rangle \neq 0 \)
\[ \mathbb{Z}_2 \]

(G) \( \langle \Phi_\mp \rangle \neq 0 \)
\[ \mathbb{Z}_2 \]

(E) \( \langle \Phi_x \rangle = 0 \)
\[ \langle \Phi_y \rangle \neq 0 \]
\[ \mathbb{Z}_2 \]

\( u_1 / |u_3| \)
\( u_2 / |u_3| \)
Broken symmetries and topological order in the Higgs phase

Phenomenologically attractive state with uni-directional CDW, Ising-nematic, and scalar spin chirality order:

$H_x = \sin \theta (0, 0, 1), \quad H_y = \cos \theta (1, i, 0)$

i.e. short-range collinear spin correlations along $x$, and short-range spiral spin correlations along $y$. 
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Gauge theory of fluctuating antiferromagnetism
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**SU(2) gauge theory:** Fractionalize the SDW order parameter into the Higgs field ($H$) and the spinons ($R$); fractionalize the electron ($c$) into chargons ($\psi$) and spinons ($R$). When the Higgs field is condensed, the $\psi$ fermions and $R$ bosons are deconfined particles in an algebraic charge liquid (ACL) state, but with a strong residual attractive interaction from the $t_{ij}$. 

Let us also note the inverse of ($5.6$) into chargons ($\psi$) and the spinons ($R$); fractionalize the electron ($c$) into chargons ($\psi$) and spinons ($R$). When the Higgs field is condensed, the $\psi$ fermions and $R$ bosons are deconfined particles in an algebraic charge liquid (ACL) state, but with a strong residual attractive interaction from the $t_{ij}$. 

Given the SU(2) gauge invariance associated with ($5.3$), $L_i$ and $S_i$ are labeled by the U(1) charge. The antiferromagnetic spin correlations are characterized by their dimension of the SU(2) representation, while those under the electromagnetic U(1) are labeled by the dimension of the SU(2) representation. This table summarizes the charges carried by the fields in the resulting SU(2) gauge theory.
Anti-ferromagnet with $p$ holes per square
ACL with density $p$ of spinless fermionic chargons $\psi$, emergent gauge fields (the blue dimers),

$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$
ACL with density $p$ of spinless fermionic chargons $\psi$, emergent gauge fields (the blue dimers),

$$\psi = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$
ACL with density $p$ of spinless fermionic chargons $\psi$, emergent gauge fields (the blue dimers),

$$= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$
ACL with density $p$ of spinless fermionic chargons $\psi$, emergent gauge fields (the blue dimers),

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ACL with density $p$ of spinless fermionic chargons $\psi$, emergent gauge fields (the blue dimers),

$$\langle \uparrow \downarrow \rangle - \langle \downarrow \uparrow \rangle \right) / \sqrt{2}$$
ACL with density $p$ of spinless fermionic chargons $\psi$, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons $R$ 

$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$
ACL with density $p$ of spinless fermionic chargons $\psi$, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons $R$

$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$
ACL with density $p$ of spinless fermionic chargons $\psi$, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons $R$.

\[ = \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}} \]
ACL with density $p$ of spinless fermionic chargons $\psi$, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons $R$

$$\hat{p}^2 = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$
ACL with density $p$ of spinless fermionic chargons $\psi$, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons $R$

$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$
ACL with density $p$ of spinless fermionic chargons $\psi$, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons $R$. 

\[ \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2} \]
Metal with electron-like quasiparticles on a Fermi surface of size $p$, and emergent gauge fields

$$\frac{p}{2} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

$$\frac{p}{2} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

Metal with electron-like quasiparticles on a Fermi surface of size $p$, and emergent gauge fields

\[
\begin{align*}
\text{FL}^* = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}
\end{align*}
\]


Metal with electron-like quasiparticles on a Fermi surface of size $p$, and emergent gauge fields.
Metal with electron-like quasiparticles on a Fermi surface of size $p$, and emergent gauge fields

\[ \text{FL*} \]

\[ = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2} \]

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\[ \text{Metal with electron-like quasiparticles on a Fermi surface of size } p, \text{ and emergent gauge fields} \]

\[ \text{Metal with electron-like quasiparticles on a Fermi surface of size } p, \text{ and emergent gauge fields} \]

\[ \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2} \]

Metal with electron-like quasiparticles on a Fermi surface of size $p$, and emergent gauge fields

\[ \frac{1}{p} = \left( \left| \uparrow \downarrow \right> - \left| \downarrow \uparrow \right> \right) / \sqrt{2} \]

\[ \frac{1}{p} = \left( \left| \uparrow \circ \right> + \left| \circ \uparrow \right> \right) / \sqrt{2} \]

Metal with electron-like quasiparticles on a Fermi surface of size $p$, and emergent gauge fields

$$FL^* = \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

$$= \frac{(|\uparrow\circ\rangle + |\circ\uparrow\rangle)}{\sqrt{2}}$$

Metal with electron-like quasiparticles on a Fermi surface of size $p$, and emergent gauge fields

$$\mathbf{FL}^*$$

\[ \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \]

\[ \frac{|\uparrow\circ\rangle + |\circ\uparrow\rangle}{\sqrt{2}} \]

In a conventional Fermi liquid state, Fermi volume must equal \((1-p) \text{ (mod 2)}\).

When the unit cell is doubled by SDW order, total Fermi volume must equal \((1-p) \text{ (mod 1)}\).

A state with Fermi volume \((-p) \text{ (mod 2)}\), but no translational symmetry breaking, must have non-quasiparticle excitations with vanishing energy on a torus i.e. emergent gauge fields (bulk topological order).

### Gauge theory of fluctuating antiferromagnetism

<table>
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<tr>
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**SU(2) gauge theory:** The ACL was used to model the photoemission and the cluster DMFT results in the intermediate phase.

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**SU(2) gauge theory for quantum criticality**: fractionalize the SDW order parameter into the Higgs field ($H$) and the spinons ($R$); but do not fractionalize the electron ($c$) into chargons ($\psi$) and spinons. In the FL$^*$ state, the low energy fermionic excitations have the quantum numbers of an electron.
Theory of optimal doping criticality

SU(2) gauge theory

SU(2) gauge theory with adjoint complex Higgs fields $H_{x,y}^a (a = 1, 2, 3)$, and gauge-invariant, electron-like fermions $c_\alpha$ with a large Fermi surface.

$$\mathcal{L} = \frac{1}{2} \left| \partial_\mu H_{x}^a - \epsilon_{abc} A_\mu^b H_{x}^c \right|^2 + \frac{1}{2} \left| \partial_\mu H_{y}^a - \epsilon_{abc} A_\mu^b H_{y}^c \right|^2$$
$$+ \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + V(H_{x,y}^a)$$
$$- \sum_{j,\rho} t_\rho \left( \bar{c}_{j,\alpha}^\dagger c_{j,\alpha} + \bar{c}_{j,\alpha}^\dagger v_{\rho,\alpha} + \bar{c}_{j,\alpha}^\dagger c_{j,\rho,\alpha} \right) - \mu \sum_j c_{j,\alpha}^\dagger c_{j,\alpha}$$
$$+ \lambda \sum_j c_{j,\alpha}^\dagger c_{j,\alpha} H^a(j) H^a(j)$$

The fermions do not have Yukawa coupling to the Higgs fields, or a minimal coupling to the gauge fields: both are prohibited by gauge invariance. We treat the quartic coupling, $\lambda$ perturbatively.
Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order


- New photoemission measurements at zero magnetic field show Fermi surfaces in quantitative agreement with quantum oscillation measurements.

- The energy gap between the electron and hole pockets collapses near \( x = 0.17 \) like an order parameter.

- “The totality of the data points to a mysterious order between \( x = 0.14 \) and \( x = 0.17 \), whose appearance favors the FS reconstruction and disappearance defines the quantum critical doping. A recent topological proposal provides an ansatz for its origin.”

Hole-doped cuprates

One or more of Ising-nematic, CDW, scalar spin chirality, and $\mathbb{Z}_2$ topological orders

Reconstructed (FL*) Fermi surfaces, with large length scale confinement in the U(1) cases

Deconfined critical SU(2) gauge theory

Fermi liquid with large Fermi surface

Higgs
Electron-doped cuprates

Reconstructed (FL*) Fermi surfaces, with large length scale confinement in a U(1) gauge theory

Fermi liquid with large Fermi surface