Stringy Excited Baryons in Holographic QCD

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Based on [arXiv: 2001.01461]
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“Applications of gauge topology, holography and string models to QCD”
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In holographic models of QCD, mesons are described by a 5 dim $U(N_f)$ gauge theory.

Baryons are often treated as solitons in the 5 dim gauge theory.

$$\#\text{baryon} = \frac{1}{8\pi^2} \int_{\Sigma_t} \text{tr} (F \wedge F)$$

→ a lot of nice results

But, this is not enough!
Pion, vector and axial vector mesons are obtained from the 5 dim gauge field:

\[ A_\mu(x^\mu, z) = \sum_{\mu = 0}^3 B^{(n)}_\mu(x^\mu) \psi_n(z) \]

\[ A_z(x^\mu, z) = \sum_{n \geq 1} \varphi^{(n)}(x^\mu) \phi_n(z) \]

\[ \varphi^{(0)} : \text{pion}, \quad B^{(1)}_\mu : \rho \text{ meson}, \quad B^{(2)}_\mu : a_1 \text{ meson}, \quad \cdots \]

\[ J^{PC} = 0^{--} \quad J^{PC} = 1^{--} \quad J^{PC} = 1^{++} \]

But, there are many other mesons found in the experiments.
$I = 1$ Mesons from PDG

blue: established
gray: not established

$J^P_C = 1^{++}, 1^{--}$

mass [GeV]

Spin, parity, charge conj.
Can we get these mesons?

We should be able to get these mesons, if the holographic QCD is really equivalent to QCD.

In fact, the top-down holographic QCD based on D4/D8 system predicts the existence of these mesons! [Imoto-Sakai-S.S. 2010]

1\textsuperscript{st} excited states

\rightarrow \ a_2(1320), \ b_1(1235), \ \pi(1300), \ a_0(1450), \ \cdots

2\textsuperscript{nd} excited states

\rightarrow \ \rho_3(1690), \ \pi_2(1670), \ \cdots

3\textsuperscript{rd} excited states

\rightarrow \ a_4(2040), \ \cdots
$I = 1$ Mesons from PDG

Furthermore,

This behavior (linear Regge trajectory) is reproduced from string theory.

\( J = \alpha_0 + \alpha' m^2 \)

\( \alpha_0 \approx 0.53 \quad \alpha' \approx 0.88 \text{ GeV}^{-2} \)

\( \rightarrow \) Clear experimental evidence of string theory!
Today,

I’d like to develop a similar story for the baryon sector.
Baryons are often treated as solitons in the 5 dim gauge theory.

$$\#\text{baryon} = \frac{1}{8\pi^2} \int \sum_t \text{tr} (F \wedge F')$$

Quantizing the light fluctuations around the soliton, a lot of baryons with \( I = J = 1/2, 3/2, \ldots \) are obtained.

For example,

\[
\begin{align*}
I = J = 1/2 : & \quad \text{n,p, N(1440), N(1535), \ldots} \\
I = J = 3/2 : & \quad \Delta(1232), \Delta(1600), \Delta(1700), \ldots
\end{align*}
\]

But, there are many baryons with \( I \neq J \).
$I = 1/2$ Baryons from PDG

blue: established
gray: not established

mass [GeV] $I=J=1/2$
$I = 3/2$ Baryons from PDG

- **blue**: established
- **gray**: not established

**Spin parity**

$\Delta I = 0$
Clearly, the previous analysis was not enough.

Question:

Can we get all these baryons from string theory?
Our proposal is to consider stringy excited states.

Baryon \sim \begin{array}{c}
\text{D-brane} \\
\text{open string}
\end{array}

\begin{array}{c}
\text{excited} \\
\text{open string}
\end{array}

(See also Cobi’s talk on June 4th)
NB:

- As you will see, there are some ambiguities both in the analysis and the interpretation.
  - I am afraid I will not be able to provide you the complete answer to the previous question. But, let me try to propose a way to answer it.

- We neglect 1/Nc and 1/λ corrections.

- Today, we only consider the Nf = 2 cases (only consider up and down quarks), and neglect the quark mass.
Plan

1. Introduction ✔
2. Holographic QCD and Baryons
3. Stringy Excited Baryons
4. Interpretation
5. Summary and outlook
(Top-down) holographic QCD

Gauge/String duality predicts the following equivalence:

4 dim SU(Nc) QCD
with Nf massless quarks
+ massive adjoint matters
( realized in a D-brane system)

Type IIA string theory
in a 10 dim curved background
with Nf probe D8-branes
“holographic QCD”

Background $\sim R^{1,3} \times R^2 \times S^4$
with RR flux
$x^\mu (y, z)$
$
\frac{1}{2\pi} \int_{S^4} dC_3 = N_c$

D8-branes $\sim R^{1,3} \times \{ z \} \times S^4$

$R^2$:

$\text{D8}$
Baryons are described as D4-branes wrapped on $S^4$. Nc open strings have to be attached, due to the RR flux.

[Witten, Gross-Ooguri 1998]

The other end point is attached on the D8-brane

$$\text{D8-branes } \sim \mathbb{R}^{1,3} \times \{z\} \times S^4$$
$$\text{D4-brane } \sim \mathbb{R} \times S^4$$

localized in the 4 dim space $\{(x^1, x^2, x^3, z)\}$

→ behaves as a particle and interpreted as a baryon
Hashimoto-lizuka-Yi “Matrix model for baryons” [Hashimoto-lizuka-Yi 2010]
(See Koji’s talk on July 30th)

Degrees of freedom living on the D4-brane:
- ground state, SO(5) invariance

D4-D4 open string $\rightarrow$ $A_0(t)$, $X^M(t)$ ($M = 1, 2, 3, z$)
- real scalar field (position of D4)
- U(1) gauge field (auxiliary)
- flavor index

D8-D4 open string $\rightarrow$ $w^I_\alpha(t)$ ($\alpha = 1, 2$, $I = 1 \sim N_f$)
- complex scalar field
- Weyl spinor of SO(4) acting on $x^M$

Lagrangian

$$L_0 = \frac{M_0}{2} \left[ \dot{X}^2 + |D_0 w|^2 - V_{\text{ADHM}}(w) - V_0(X, w) \right] + N_c A_0$$

$$D_0 w = \dot{w} - i A_0 w$$

$$V_{\text{ADHM}}(w) = c \left( \text{tr}(\bar{w} w^\dagger) \right)^2$$

$$V_0(w, X) = \frac{2}{3} (X^z)^2 + \frac{1}{6} |w|^2$$

$$M_0 = \frac{\lambda N_c}{27 \pi}, \quad c = \frac{\lambda^2}{36 \pi^2}$$
• **Comments**

• Baryon states are obtained by quantizing this system.

• If one restricts $w$ to be at the bottom of $V_{ADHM}(w)$,  
  $\{X^M, w^I_\alpha\}$ parametrize the instanton moduli space. 
  (ADHM construction) → agrees with the soliton approach.

• Then, the baryons with $I=J$ obtained in the soliton approach are reproduced.

• Recently, Hashimoto-Matsuo-Morita studied this system (including $N_f>2$ and multi-baryon cases) in detail regarding $V_{ADHM}(w)$ as a perturbation, and found baryons with $I\neq J$ in the spectrum.  (→ Koji’s talk on July 30\textsuperscript{th})
Our main idea is to keep the massive modes:

- D4-D4 open string $\rightarrow A_0(t), \quad X^M(t), \quad \Phi_k(t) \quad (k = 1, 2, \cdots)$
- D8-D4 open string $\rightarrow w^I_\alpha(t), \quad \Psi_j(t) \quad (j = 1, 2, \cdots)$

Quantizing these open strings, we obtain

\[
\begin{array}{c|c|c|c}
N_{44} & \text{spin} & \text{parity} \\
\hline
1 & 2 \oplus 1 \oplus (0 \times 3) & + \\
& 1 \oplus 0 & - \\
\end{array}
\]

\[
\begin{array}{c|c|c|c}
N_{84} & \text{spin} & \text{parity} \\
\hline
\frac{1}{2} & \frac{3}{2} \oplus \frac{1}{2} & - \\
1 & \frac{5}{2} \oplus \left(\frac{3}{2} \times 2\right) \oplus \left(\frac{1}{2} \times 3\right) & + \\
\end{array}
\]
Lagrangian

\[ L = L_0 + L_m \]

\[ L_m = \frac{M_0}{2} \left[ \sum_j (|D_0 \psi_j|^2 - m_j^2 |\psi_j|^2) + \sum_k (\Phi_k^2 - m_k^2 \Phi_k^2) + L_{\text{int}} \right] \]

\[ m_j^2 = \frac{N_{84}}{\alpha'} \quad (N_{84} = 1/2, 1, 3/2 \cdots) \]

\[ m_k^2 = \frac{N_{44}}{\alpha'} \quad (N_{44} = 1, 2, \cdots) \quad \alpha' = \frac{27}{4\lambda} \]

In the following we neglect \( L_{\text{int}} \), hoping that its contribution is small.
Quantization

- We work in the $A_0=0$ gauge and impose the Gauss law constraint on the Hilbert space of physical states. [Hashimoto-Matsuo-Morita 2019]

- Gauss law equation (EOM for $A_0$)

$$q_w + \sum_j q_j = N_c$$

$$q_w \equiv \frac{M_0}{2} \text{tr}(i(w^\dagger w - w^\dagger w)) , \quad q_j \equiv \frac{M_0}{2} i(\psi_j^\dagger \psi_j - \psi_j^\dagger \psi_j)$$

charges associated with the phase rotation of $w$ and $\Psi_j$

$q_j \sim$ number of open strings in state $j$.

Gauss law eq. $\rightarrow$ total number of strings is $N_c$

- We consider the cases with

$$q_w \sim \mathcal{O}(N_c), \quad q_j \sim \mathcal{O}(1)$$
It is convenient to parametrize \( w = (w^I_{\alpha}) \) as

\[
    w = Y_0 1_2 + i\vec{Y} \cdot \vec{\tau} \quad (Y = (Y_0, \vec{Y}) \in \mathbb{C}^4)
\]

\[
    Y = e^{i\theta}(y + i\tilde{y}) \quad \tilde{y} = \tilde{\beta} i\vec{\Sigma} y/\rho, \quad \rho \equiv \sqrt{y^2}
\]

\[
    (y \in \mathbb{R}^4, \tilde{\beta} \in \mathbb{R}^3)
\]

\( \vec{\Sigma} \): the generators of \( SU(2)_I = SU(N_f) \) (for \( N_f=2 \)) embedded in \( SO(4) = SU(2)_J \times SU(2)_I \) acting on \( Y \).

spin isospin

\[
    w \in \mathbb{C}^4 \rightarrow (y, e^{i\theta}, \beta_a) \in (\mathbb{R}^4 \times U(1))/\mathbb{Z}_2 \times \mathbb{R}^3
\]

with the identification \( \mathbb{Z}_2 : (y, e^{i\theta}) \rightarrow (-y, -e^{i\theta}) \)

Then

\[
\begin{align*}
    qw &= -i \frac{\partial}{\partial \theta} \\
    V_{\text{ADHM}}(w) &= 16c\rho^2\beta^2
\end{align*}
\]
Note that the kinetic terms in the Hamiltonian contain

\[
\frac{1}{2M_0} |P_w|^2 = \frac{1}{2M_0 \rho^2} \frac{\partial^2}{\partial \theta^2} + \cdots
\]

\[
= \frac{q_w^2}{2M_0 \rho^2} + \cdots
\]

This term acts as a potential for \( \rho \)

Including this term, the minimum of the potential is

\[
\rho^2 = \frac{\sqrt{6} |q_w|}{2M_0} \equiv \rho_0^2
\]

which is of order \( 1/\lambda \).
At the leading order in the $1/N_c$ and $1/\lambda$ expansions, the Hamiltonian becomes

\[
H_0 \simeq \frac{1}{2M_0} P_X^2 + \frac{M_0}{3} (X^z)^2 + \frac{M_0}{3} \rho_0^2
\]
\[- \frac{1}{4M_0} \left[ \left( \frac{\partial^2}{\partial(\delta \rho)} \right)^2 + \left( \frac{\partial}{\partial \beta} \right)^2 \right] + M_0 \left( \frac{2}{3} \delta \rho^2 + \omega_\beta^2 \beta^2 \right)
\]

\[
H_m = \sum_j \left( \frac{1}{2M_0} |P_{\psi_j}|^2 + \frac{1}{2} M_0 m_j^2 |\psi_j|^2 \right) + \sum_k \left( P_{\phi_k}^2 + \frac{1}{2} M_0 m_k^2 \phi_k^2 \right)
\]

where \( \delta \rho \equiv \rho - \rho_0 \), \( \omega_\beta^2 \equiv 8c_0^2 \rho_0^2 \)

This is just a collection of harmonic oscillators and we obtain the mass formula:

\[
M \simeq M_0^* + \sqrt{\frac{2}{3}} (n_z + n_\rho) + \omega_\beta \sum_{a=1}^3 n_a^\beta + \sum_j m_j (n_j^\psi + n_j^\overline{\psi}) + \sum_k m_k n_k^\phi
\]

\[n_z, n_\rho, n_\beta^a, n_j^\psi, \overline{n_j^\psi}, n_k^\phi \in \mathbb{Z}_{\geq 1}\]
Comments

- Note that \( \omega_\beta^2 = O(\lambda) \), which is comparable to the mass squared of the massive modes. \( \beta \) becomes heavy because of \( V_{\text{ADHM}}(w) \).

- The neglected interaction term \( L_{\text{int}} \) may contribute to generate mass shifts for the massive modes.

- The first term in the mass formula is

\[
M_0^* = \left( 1 + \frac{\rho_0^2}{3} \right) M_0 + \text{(zero point energy)}
\]

We don’t know how to compute the zero point energy. 
→ Leave it as an unknown parameter.
4 Interpretation

Regge trajectory

Consider the following states

mass [GeV]  \[ I = \frac{1}{2} \] Baryons from PDG

Spin parity  \[ JP \]
Spin $J$ as a function of mass$^2$

People have considered this as an evidence that these states are given by [Sharov 2013, Sonnenschein-Weissman 2014 etc]
See also quark-diquark model: [Santopinto 2004, Gutierrez-Sanctis 2009, Ferretti-Vassallo-Santopinto 2011, Scnctis 2014, etc.]

one of the open string is excited (rotating)
Let’s assume that this is correct and see what we get.

Our mass formula suggests

\[ M = M_0^*|_{q_w = N_c - 1} + \frac{1}{\sqrt{2\alpha'}} \sqrt{J - \frac{1}{2}} \quad (J \geq 3/2) \]

which gives

with \( \alpha' \simeq 0.6 \text{ GeV}^{-2} \), \( M_0^* \simeq 0.5 \text{ GeV} \)

cf) For \( \rho \) trajectory: \( \alpha' \simeq 0.88 \text{ GeV}^{-2} \)

Using the value of \( \lambda \) and \( M_{KK} \) to fit \( m_\rho \) & \( f_\pi \): \( \alpha' \simeq 0.45 \text{ GeV}^{-2} \)
Other baryons

- Additional information:
  - kinetic term for \( y \in \mathbb{R}^4 \) contains \( \text{Laplacian for } S^3 \)

\[
- \frac{1}{4M_0} \left( \frac{\partial}{\partial y_A} \right)^2 = - \frac{1}{4M_0} \left( \frac{1}{\rho^3} \partial_\rho (\rho^2 \partial_\rho) + \frac{1}{\rho^2} \Delta_{S^3} \right)
\]

The eigenvalue of \( \Delta_{S^3} \) is \(-\ell(\ell + 2), \ (\ell = 0, 1, 2, \cdots)\)

This is neglected in the previous formula, because it is subleading in \(1/\text{Nc}\). But, we expect larger \(\ell\) gives heavier states

- This part contributes to isospin/spin by \( I = J = \ell/2 \)

- The identification \( \mathbb{Z}_2 : (y, e^{i\theta}) \rightarrow (-y, -e^{i\theta}) \) implies \( \ell \equiv q_w \text{ (mod 2)} \).
\( I = \frac{1}{2} \)

<table>
<thead>
<tr>
<th>level</th>
<th>( \ell )</th>
<th>( q_{\omega} )</th>
<th>( J^P )</th>
</tr>
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<tbody>
<tr>
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<td>( N_c - 1 )</td>
<td>( 3/2^- )</td>
</tr>
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<td></td>
<td>2</td>
<td>( N_c - 1 )</td>
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<td>( N_c )</td>
<td>( \frac{5}{2}^+ \oplus \left( \frac{3}{2}^+ \times 2 \right) \oplus \left( \frac{1}{2}^+ \times 4 \right) \oplus \frac{3}{2}^- \oplus \left( \frac{1}{2}^- \times 2 \right) )</td>
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**I = 1/2 Baryons from PDG**

![Diagram of mass vs. spin-parity for I = 1/2 baryons from PDG]
### $l=1/2$

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### Figure

**I = 1/2 Baryons from PDG**

mass [GeV]

$I=J=1/2$
We studied stringy excited baryons in holographic QCD.

We developed a formalism to include excited modes for the open strings attached on a D4-brane corresponding to a baryon.

A lot of states are obtained.

How can we get more convincing correspondence between predicted states and observed baryons?

- The effect of $L_{\text{int}}$?
- $1/N_c$ and $1/\lambda$ corrections?

Generalization to $N_f = 3$, $\#\text{baryon}>1$ would be interesting.
Thank you!